

clear the current image (option A). Now repeat the steps in §17.2.2⁵, but use the equatorial-coordinate *g*-band source list, rather than creating a new one. This will ensure that you measure the same stars in the same order in both filters. If you didn't, then we would not easily be able to make the color-magnitude diagram for the cluster in the next section.

17.3 Color-Magnitude Diagrams

17.3.1 Introduction

17.3.2 Constructing the Diagram

Now that you have lists of the cluster's star magnitudes in two filters, we'll use those to make the color-magnitude diagram. Open your Excel or another spreadsheet program and create a new, blank form. Open the *g*-band photometry table (or whichever filter is the shorter wavelength of your pair) in Excel and copy the **Number** and **Magnitude** columns into the first two columns of your new form. Retitle **Number** as **g Number** and **Magnitude** as **g**, because it is standard (and convenient) to represent apparent magnitudes with the filter name. Now do the same for the *r*-band table, putting them in the third and fourth columns. Retitle its number as **r Number** and its magnitude as **r**.

Look at the source numbers for the *g* and *r* tables. Ideally, they will match up, line for line, all the way to the bottom. But if the *r* image is shifted with respect to the *g* image, then some of the sources found in *g* may not show up in *r*, creating a mismatch. Everywhere you see numbers skipped in the *r* list, select the **r Number** and **r** cells for the next source and have Excel **Insert Cells**, shifting the selected cells down (Fig. 17.4).

We should not leave the magnitudes blank, even for the rows with missing stars, but Excel has a handy function for this that identifies something as “Not A Number.” You may see this as “NaN” in some programs. In Excel, go into each blank cell in the magnitude column and enter `=NA()`. Once

⁵**Note:** In earlier versions of APT, I have seen the list processing hang for hours without ever completing. I believe this was caused by it using up too much memory while processing the second filter. If that happens to you, then you should quit APT after the first filter and restart the program before running the second filter. I have not seen this bug in newer versions.

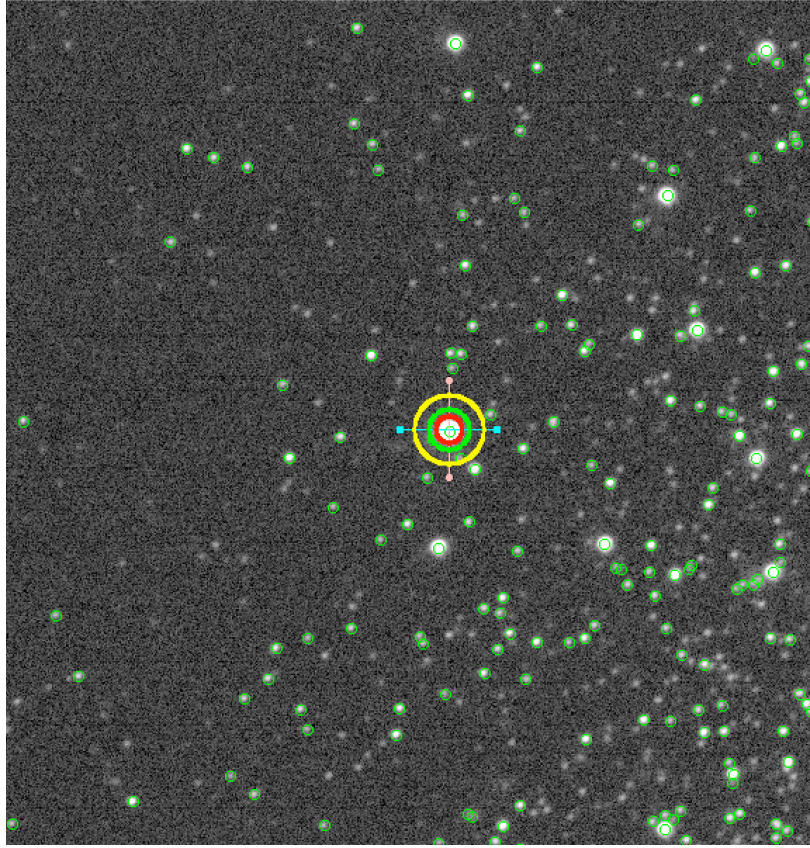


Figure 17.3: When you choose to overlay the sources on the image, APT draws green circles around each star it has identified. If the circles are centered on the stars, then all is going well. If they are systematically offset from the stars, then you're using a pixel-coordinate list from an image that's shifted with respect to this one. Create an equatorial-coordinate list, instead. The bullseye is the aperture around a star I clicked on before processing the list.

	A	B	C	D		A	B	C	D
1	g Number	g	r Number	r		g Number	g	r Number	r
2	1	19.5752	3	19.6369		1	19.5752		
3	2	20.5935	4	19.9823		2	20.5935		
4	3	19.4917	5	20.2709		3	19.4917	3	19.6369
5	4	20.0597	6	17.0372		4	20.0597	4	19.9823
6	5	20.2172	7	21.0465		5	20.2172	5	20.2709
7	6	17.4312	8	19.5831		6	17.4312	6	17.0372
8	7	20.6904	9	21.1973		7	20.6904	7	21.0465

Figure 17.4: Inserting a block of cells to make the g and r lists match. Since two sources are missing from r , this procedure was done twice.

	A	B	C	D		A	B	C	D
1	g Number	g	r Number	r		g Number	g	r Number	r
2	1	19.5752		=NA()		1	19.5752		#N/A
3	2	20.5935				2	20.5935		#N/A
4	3	19.4917	3	19.6369		3	19.4917	3	19.6369
5	4	20.0597	4	19.9823		4	20.0597	4	19.9823
6	5	20.2172	5	20.2709		5	20.2172	5	20.2709
7	6	17.4312	6	17.0372		6	17.4312	6	17.0372
8	7	20.6904	7	21.0465		7	20.6904	7	21.0465

Figure 17.5: Setting the magnitudes of missed sources to #NA

entered, it will be displayed as #NA for “number not available” (Fig. 17.5). You can also type #NA directly into the cell.

Now we are ready to calculate the color of each star, which is the difference between the g and r magnitudes. Label the next column $g-r$, and enter that formula in the cells underneath it. Note how any stars with #NA listed for a magnitude will not cause an error when Excel calculates the difference. Rather, the color is listed as #NA, itself. Finally, copy the g -band magnitudes to the column to the right of the color, so we can use it as the vertical axis of our plot (Fig. 17.6).

	A	B	C	D	E	F	G
1	g Number	g	r Number	r		g-r	g
2	1	19.5752		#N/A		=B2-D2	19.5752
3	2	20.5935		#N/A		#N/A	20.5935
4	3	19.4917	3	19.6369		-0.1452	19.4917
5	4	20.0597	4	19.9823		0.0774	20.0597
6	5	20.2172	5	20.2709		-0.0537	20.2172
7	6	17.4312	6	17.0372		0.394	17.4312
8	7	20.6904	7	21.0465		-0.3561	20.6904

Figure 17.6: The data for our plot comes from columns F and G in this example: color ($g - r$) and magnitude (g).

At this point, we're ready to create the color-magnitude diagram itself. Select the **g-r** and **g** columns and create a marked scatter plot without connecting lines (Fig. 17.6). This, technically, is the color-magnitude diagram, but we can't make much of it in Excel's default form. Take the following steps to clean it up in a way that we can interpret it:

- Shrink the markers (the points) so that you can see the patterns. I like to make them 2-3 pt. and remove the default shadow, which makes it harder to tell where the point actually lies.
- The brighter stars must be at the upper end, but for magnitudes that means lower numbers. So put the vertical axis values in reverse order.
- Crop the vertical axis (magnitude) range to fit your data. In my example, that's about $14 < g < 21$, but it will vary for other clusters. The bright end (minimum magnitude) will be lower for closer clusters. For SDSS data, the faintest (largest magnitude) you'll ever get is around 23rd magnitude, and those are usually not reliable. For other observatories, the faintest end will depend on the exposure time.
- Crop the horizontal axis (color) range to run about $-0.5 < (g-r) < 1$. You'll see a few scattered stars lying outside this range, but for most clusters, the stars actually belonging to the cluster and producing the distinctive patterns we'll look for will lie close to this range.

In addition to these important steps, you can make the diagram look better by removing the "legend" and grid lines and by adding axis labels and a title. It is common to make color-magnitude diagrams rather narrow, as I've done in the right-hand example in Fig. 17.7. This keeps the Main Sequence and the Red Giant Branch as clear lines.

17.4 Finding Age and Distance

Compare the Hertzsprung-Russell Diagram here (Fig. 17.8) with the Color-Magnitude Diagram (CMD) that we've made. The vertical axis is the logarithm of luminosity, while the CMD has magnitude, but these wind up looking the same. The horizontal axis is temperature, while our CMD has color. Color gives us temperature information, but it scales a bit differently. In a CMD, we may see our Main Sequence and Red-Giant Branch tilted nearly vertically.

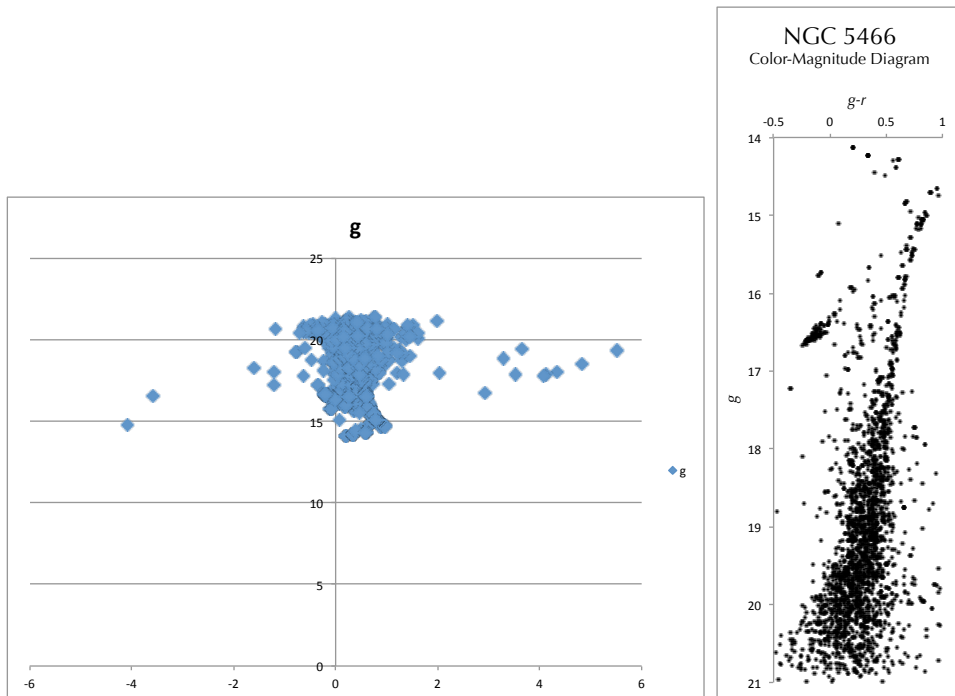


Figure 17.7: The data for our plot comes from columns F and G in this example: color ($g - r$) and magnitude (g). The default settings produce the version on the left. After some adjustments, we can get the nice-looking and recognizable color-magnitude diagram on the right.

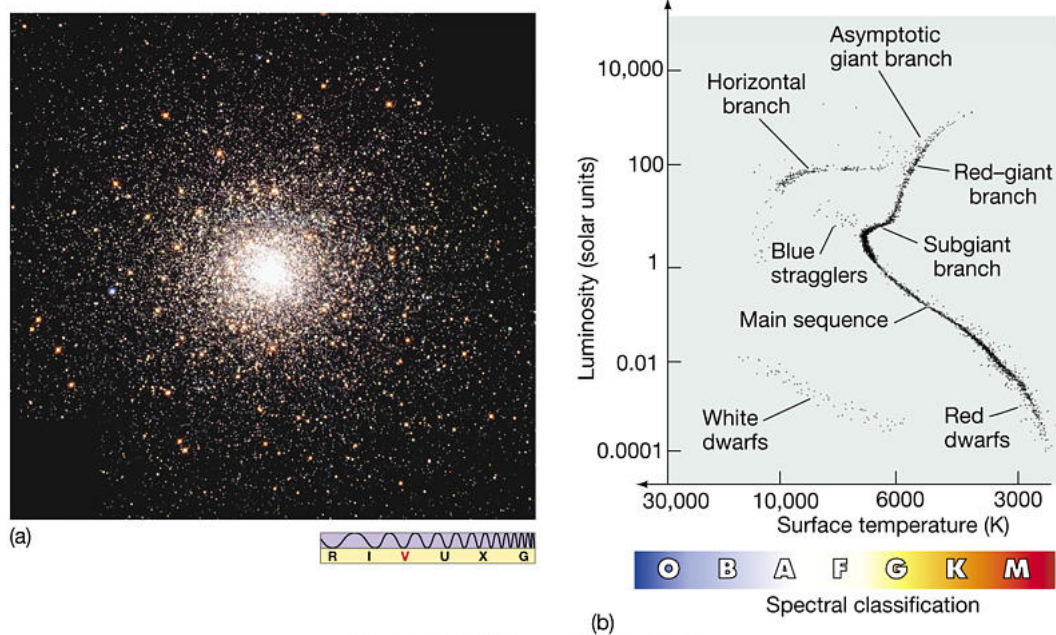


Figure 17.8: Schematic of a Hertzsprung-Russell Diagram, from *Astronomy Today*.

17.4.1 The Rough Approach: Using the Horizontal Branch

We want to measure the age of and distance to the cluster. We can get a rough idea by looking at the Horizontal Branch stars. The Horizontal Branch has nearly the same absolute magnitude (M) from one globular cluster to another. In the g band, it's about $M_g \sim +0.6$, based on my survey of several clusters in the literature. If we measure the apparent g magnitude of our cluster, we can calculate the “distance modulus,” which is the difference between the apparent and absolute magnitudes, $m - M$ (see §5.3.4). From the equations in that section, we find the distance in parsecs to be

$$d = 10^{\frac{5+m-M}{5}} \text{ pc} , \quad (17.1)$$

where m and M are the apparent and absolute magnitudes, respectively, in whichever filter we're using. So if our Horizontal Branch apparent magnitude is $g = 16.0$, and its absolute magnitude is $M_g = +0.6$, then the distance modulus is $g - M_g = 15.4$. Subbing that into the distance equation, we would get $d = 12,023$ pc, or $d = 12$ kpc.

17.4.2 The Sophisticated Approach: Fitting a Model Isochrone

Obtaining Isochrones

As it happens, there are some differences in the absolute magnitudes of the Horizontal Branch stars in different clusters, and we can do a more careful job by fitting a theoretical model of the cluster's stars to its color-magnitude diagram. This computer model is called an *isochrone*, Greek for “same time,” or “same age,” because it's a model in which all of the simulated stars are the same age, as we expect for a cluster. In practice, it's a table of simulated stars, one star per row, where the table columns give the star's absolute magnitude in different filters.

If you are doing this as part of my Observational Astronomy course, a selection of isochrones have been uploaded to Blackboard for you to use. The tables have the $g - r$ color and g magnitude and are ready to open in Excel.

For anyone not in my class, you can download these yourself from archives on the web, such as Léo Girardi's CMD (Color-Magnitude Diagram)

Observational Astronomy students can skip over the following steps!

tool at the Osservatorio Astronomico di Padova:

`stev.oapd.inaf.it/cgi-bin/cmd`.

There are a lot of options here, but you only need to decide on a few.

1. Under **Photometric System**, choose the filter set we're using: SDSS u, g, r, i, z , or whatever is appropriate for the telescope you're gotten the images from. The model will include magnitudes for every filter in the set.
2. Under **Ages/Metallicities**, select **Linear Age** and enter the desired age in years under **initial value**. Since these can be billions of years old, it's useful to enter the age in scientific notation, but note that the computer style of scientific notation is a bit different: 12.4×10^9 becomes **12.4e9**. The **e** marks the exponential part, and there should be no spaces. By leaving **step=0**, you get a single age out and don't have to worry about the **final value**. Use ages in the range from about 1–14 Gyr (**1e9** to **14e9** yr), because globular clusters are very old.
3. “Metallicity” is a measure of how much “metal” content the star has, relative to the Sun. I've put metal in scare quotes, because to an astronomer, any element heavier than helium is called a metal. The average metallicity of the clusters I listed in this chapter is $[M/H] = -1.8$, so select the second option and set the **initial value** to -1.8.
4. Take the default settings on everything else.
5. Click **Submit**. A new page will load with a link to your table. The table is a text file, with columns separated by spaces. Save it with a filename that includes the age you specified.
6. Open it in Excel. Import starting with row 12, which will skip over most of the header information and leave the column headings.
7. Treat it as a **Delimited** file (that is, columns are separated by some symbol), with **whitespace** and **tabs** as the delimiters. Treat consecutive delimiters as one.
8. Because the column header row starts with **#** and a space, Excel puts that in a separate column, and the column headers are shifted one cell

to the right. So delete the # cell in the first row, shifting cells to the left. Look for the **gmag** and **rmag** columns. They contain the g and r absolute magnitudes.

9. Calculate $g - r$, and delete everything except two columns: **g-r** and **g**, in that order. Save this as a new spreadsheet, with the isochrone age in the filename.

Fitting Isochrones

Once you have several isochrone files and a color-magnitude diagram for your cluster, you're ready to fit the models. Look at the differences in the model CMDs shown in Fig. 17.9. Pick an isochrone age that looks close to what your CMD shows. In Excel, copy the $g - r$ and g columns from the isochrone onto your cluster's spreadsheet. Before we overplot them on the color-magnitude diagram, we need to make two adjustments:

(1) The most important is that we need to convert the absolute magnitudes of the isochrone into apparent magnitudes, using the distance modulus, $m - M$. Label a cell for the distance modulus and put in 0 as a starting value.

(2) Next, we will account for dust in the Milky Way that dims and reddens the light from this cluster. This effect is called "extinction," because it extinguishes the light, and it is labeled A_F , where F is the name of the filter. Go to the NASA/IPAC Extragalactic Database, NED (ned.ipac.caltech.edu), and put the cluster name into the search field. On the cluster's page, you have several tabs for different pieces of information. What we want now is the **Galaxy Extinctions**. Under that tab, you'll scroll down to find the filters you are using. For NGC 5466, we find the Galactic extinctions in the SDSS g and r filters are $A_g = 0.057$ and $A_r = 0.040$ magnitudes. That is, Galactic dust dims its light in the g filter by 0.057 magnitudes. Create cells for A_g , A_r , and one more for the difference between them, $A_g - A_r$. Note that we subtract them in the same order as $g - r$.

Now we'll apply the dust extinction and distance modulus. Let's consider the unmodified color and magnitude to be the raw model. First we'll add the differential dust extinction ($A_g - A_r$) to the color term⁶. Extinction is always *added* to a magnitude. Create a new column, **g-r adjusted**, and enter the function $= (g - r) + (A_g - A_r)$, where you put the corresponding cells in for the variables. For the cell containing $(A_g - A_r)$, make sure you

⁶Note that $(g + A_g) - (r + A_r) = (g - r) + (A_g - A_r)$.

Observational Astronomy students should continue here.

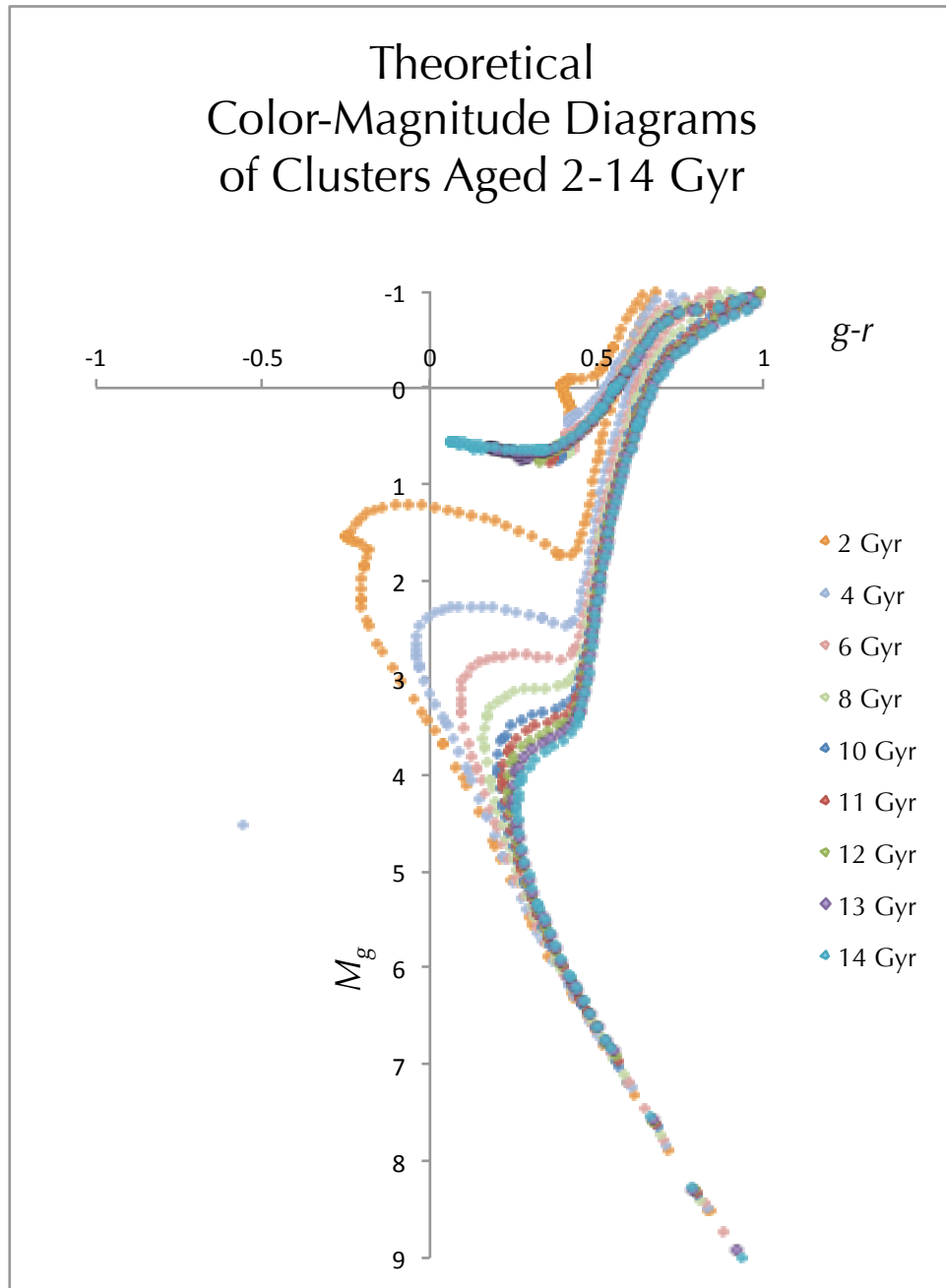


Figure 17.9: Comparison of CMDs for isochrones ranging in age from 2–14 Gyr, with metallicity $[M/H] = -1.8$, the average of the clusters listed in this chapter. Note how the Main Sequence Turnoff Point and the Subgiant Branch connected to it show the largest differences. The Red Giant Branch and Asymptotic Giant Branch show slight changes. The Horizontal Branch stays nearly perfectly fixed over this wide range of ages.

I	J	K	L	M	N	O	P
13.57 Gyr							
Model (Raw)		Adjustments			Model (Adjusted)		
g-r	g	Extinction			g-r	g	
2.128	16.374				=I4+\$M\$7		32.231
1.979	15.234	A_g	0.057		1.996		31.091
1.915	14.856	A_r	0.04		1.932		30.713
1.687	13.605	A_g - A_r	0.017		1.704		29.462

I	J	K	L	M	N	O	P	Q
13.57 Gyr								
Model (Raw)		Adjustments			Model (Adjusted)			
g-r	g	Extinction			g-r	g		
2.128	16.374				2.145	=I4+\$M\$5+\$M\$10		
1.979	15.234	A_g	0.057		1.996		31.091	
1.915	14.856	A_r	0.04		1.932		30.713	
1.687	13.605	A_g - A_r	0.017		1.704		29.462	
1.627	13.264	Distance Modulus			1.644		29.121	
1.561	12.907				1.578		28.764	
1.473	12.397	m-M	15.8		1.49		28.254	
1.357	11.658	d	14,454 pc		1.374		27.515	

Figure 17.10: Adjust the model's raw color and magnitude by adding the extinction terms and the distance modulus.

put the \$ symbol before the letter and number, so that it will stay fixed. In my example, the first cell of the raw $g - r$ is I4, and $(A_g - A_r)$ is in M7, so this comes out =I4+\$M\$7 (Fig. 17.10, top).

Next, we'll add the A_g extinction and the distance modulus to the raw model's absolute g magnitude. Create a new column, g adjusted, and enter the function $= g + A_g + DM$, where DM is the distance modulus. Don't forget to insert the \$'s on both A_g and DM . In this example, this becomes =J4+\$M\$5+\$M\$10 (Fig. 17.10, bottom).

With the adjusted $(g - r)$ and g , we can plot the isochrone in the same graph as the data and make a fair comparison. To overplot the isochrone, go to the graph and choose **Select Data**. Add another data series and choose the X as the adjusted $g - r$ and Y as the adjusted g . With the distance modulus set at 0, the graph will look like the left graph in Fig. 17.11. Adjust the distance modulus until you get the isochrone to match the real data reasonably well. The Horizontal Branch is the clearest feature to match, and the Red Giant Branch is visible even in distant, faint clusters.

With a good fit on the distance modulus, now copy in the $g - r$ and g columns from an isochrone of another age into the raw data columns on your spreadsheet. Find their best-fitting distance modulus and compare the results between the two fits. Choose the isochrone with the age that fits the best. Compare the center and right graphs in Fig. 17.11. The center graph is the better of the two, so we conclude that NGC 5466 has an age of about 14

Gyr and a distance modulus of 15.8 magnitudes. Of course, we need to try several isochrones of different ages to know what’s really the best fit, rather than just doing one old and one young model.

To determine the actual distance, we’ll use Equ. 17.1. From the distance modulus of 15.8 mag, we get a distance of 14,500 pc, or 14.5 kpc. Congratulations! You have done a fairly sophisticated job of measuring a globular cluster’s age and distance.

17.5 Mapping the Milky Way

Next, we’ll do the same for several clusters and use it to map out the Milky Way Galaxy. Knowing a cluster’s sky coordinates (which can simply be looked up) and its distance (which you measured), you know where it is in 3-D space. Since the globular clusters are known to surround the Milky Way, we can get an idea of the Milky Way’s size and our position within it by plotting these in a spreadsheet. Because the Milky Way is a spiral galaxy, it has the shape of a flattened disk and a natural plane of symmetry, the “Galactic Equator,” and perpendicular to the disk are the Galactic Poles. It will be convenient to use their Galactic coordinates and plot their locations face-on to the Galactic plane. The Galactic coordinate system uses l for Galactic longitude and b for Galactic latitude, as shown in Fig. 17.12.

We’ll plot them in x, y, z Cartesian coordinates, and given the flat nature of the Galactic⁷ disk, it’s sensible to align the x, y plane with the plane of the disk. This will all be easiest if we use Galactic system for the sky coordinates:

$$x = d \cos(l) \cos(b),$$

$$y = d \sin(l) \cos(b),$$

$$z = d \sin(b),$$

where d is the distance you calculated in Equ. 17.1 (kpc will be convenient units), and l & b are Galactic longitude and latitude, respectively, measured in degrees. Note from the form of the equations that whatever units you use for d will be the units you get for x, y , and z . The x - y plane is the plane of the Galactic disk, and the z axis is perpendicular to it. The $+x$ -axis will point towards the Galactic Center and the $+z$ axis at the Galactic North

⁷When we capitalize “Galactic,” it always refers to *our* galaxy

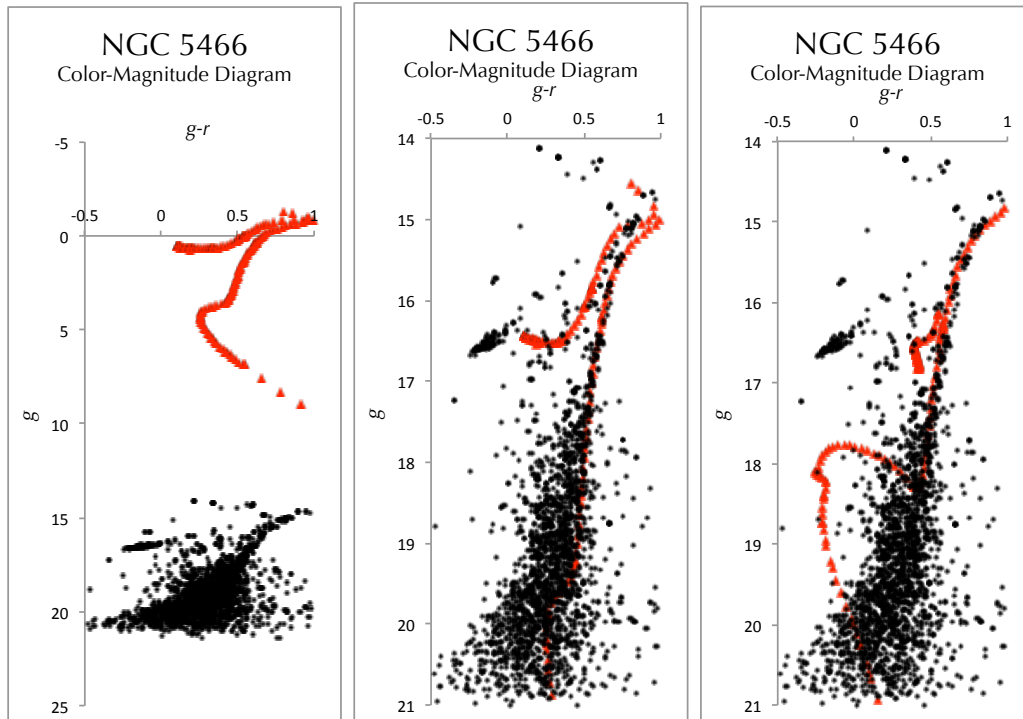


Figure 17.11: CMD of the cluster (black) and an isochrone (red). **Left:** 14 Gyr isochrone with distance modulus 0 mag (no correction for distance). **Center:** 14 Gyr isochrone with distance modulus 15.8 mag. **Right:** 2 Gyr isochrone with distance modulus 16.5 mag. I adjusted the distance modulus in each case until the Red Giant Branch fitted as best it could and the Horizontal Branch looked level with the Blue Clump stars in the cluster (NGC 5466 doesn't have a complete Horizontal Branch). Note how we determine the age by how closely the isochrone matches the shape of the color-magnitude diagram. The center is the best fit, so we determine the age is about 14 Gyr and the distance modulus is 15.8 mag.

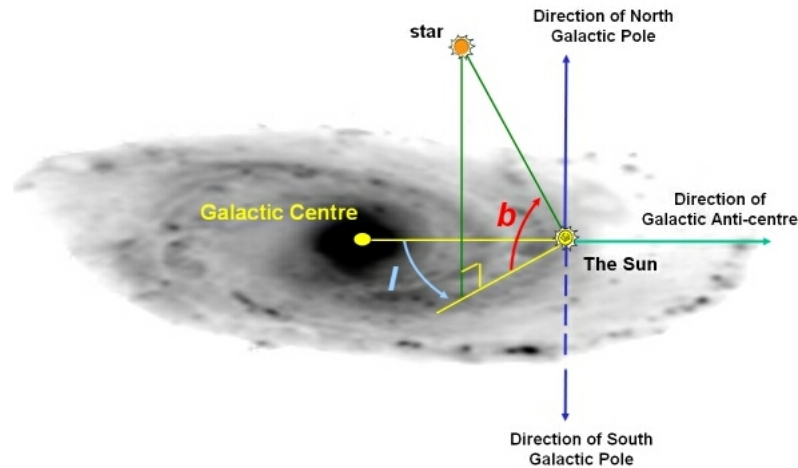


Figure 17.12: Galactic coordinate system: longitude (l) gives angle eastward (+) or westward (-) from the Galactic Center; latitude (b) gives angle northward (+) or southward (-) from the Galactic Plane. (c) Swinburne University astronomy.swin.edu.au

Pole. The way I have set it up here, we (our Solar System) will be at the origin. We will make a scatter plot of the x and y coordinates and not worry about the z values.

To calculate the (x, y) coordinates in a spreadsheet, be aware that the trigonometric functions in spreadsheets often take the angle in *radians*, rather than degrees. Check the instructions for your spreadsheet. If it uses radians, you should calculate the x and y this way:

$$x = d * \text{COS}(l * \text{PI}() / 180) * \text{COS}(b * \text{PI}() / 180)$$

$$y = d * \text{COS}(l * \text{PI}() / 180) * \text{SIN}(b * \text{PI}() / 180),$$

where $\text{PI}()$ is Excel's function for π , and we convert from degrees to radians using the conversion

$$\pi \text{ rad} = 180^\circ.$$

Where do we find a cluster's Galactic coordinates? Go back to the NED database, mentioned earlier, and after you enter the cluster name, select the **Coordinates** tab. It will give you the Galactic coordinates in degrees, as well as coordinates in many other systems we don't need right now. Enter the coordinates and your measured distance, and plot the position as a 2-D scatter plot, such as shown in Fig. 17.13.

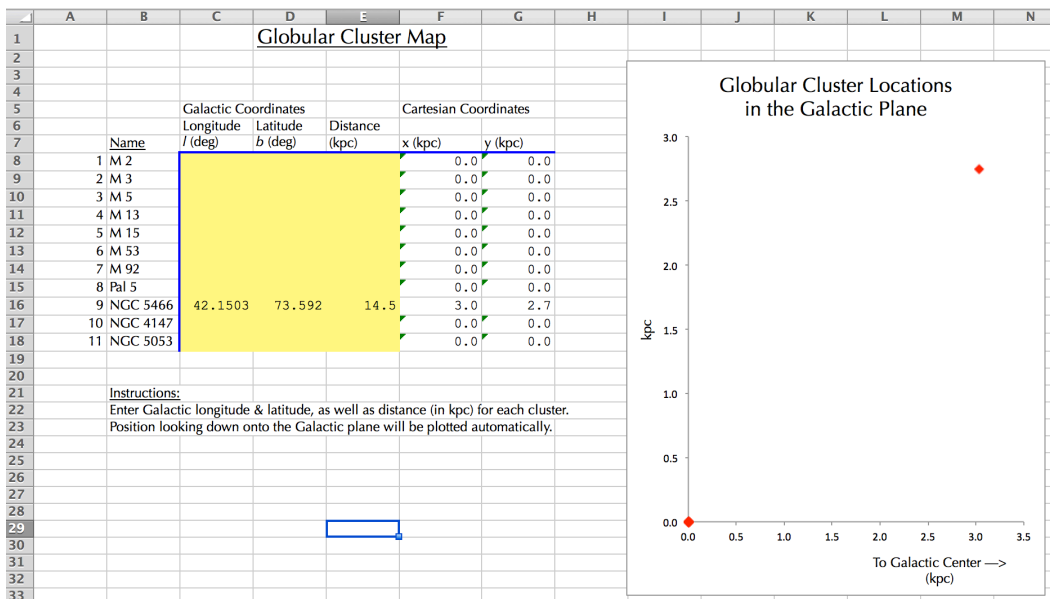


Figure 17.13: Sample spreadsheet for calculating x, y coordinates of the globular clusters and plotting them across the Galactic plane. NGC 5466 is shown as an example.

17.6 Lab Report

Your lab report should include:

- All 11 globular clusters (if you don't have all of them done, some points will be taken off for incomplete work, but go on and write up the lab report based on what you do have).
- For each cluster, you should include the color-magnitude diagram showing the cluster data and the best-fitting isochrone together.
- A single table listing each cluster, its age, and its distance.
- A plot showing the locations of the clusters around the Milky Way (from the template spreadsheet I made for you). I only want the plot, not the rest of this spreadsheet.
- Assuming that the Milky Way is roughly circular, even if the clusters are spread across it in an irregular pattern, figure the width of the Milky Way and how far we are from the center. In practice, I see that the clusters largely stretch out in a line. I believe that this happens because the SDSS only covers a section of the sky, and this is where that view intersects the Milky Way. Try to draw a circle (by hand is fine) that encloses all of the clusters you have plotted.
 - How wide is that circle? That is your measurement of the width of the Milky Way.
 - How far are we from the center and the edge of that circle? That gives our position within the Milky Way.

As far as the sample selection:

- I chose globular clusters because they lie above and below the plane of our Galaxy, so they are not much hidden by dust, and we can see them farther away. Therefore, we can use them to map out the extent of the Milky Way. These globular clusters are those that are:
 - (a) within the field of view of the SDSS
 - (b) close enough to see the main sequence and obtain a measurement of both distance and age.

- Because of point (b), you will probably be underestimating the true size of the Milky Way by a bit.