

Chapter 15

Measuring exoplanets with the transit method

15.1 Background

When a planet crosses in front of or “transits” its host star, it blocks a tiny bit of its light. While we can’t see the shadow of the planet (even the star is less than a pixel across in our cameras), we can see the starlight dim.

15.1.1 Measuring the Distance from the Star to the Planet

Recall Kepler’s Third Law,

$$P^2 \propto a^3,$$

where P is the planet’s orbital period, and a is its semi-major axis (for a circular orbit, this will be the orbital radius).

$$\frac{a^3}{P^2} = \frac{G(\mathcal{M}_* + \mathcal{M}_p)}{4\pi^2},$$

where \mathcal{M}_* is the mass of the host star, \mathcal{M}_p is the mass of the planet, and G is Newton’s gravitational constant.

If $\mathcal{M}_p \ll \mathcal{M}_*$ (the star is much more massive than the planet), then $\mathcal{M}_* + \mathcal{M}_p \approx \mathcal{M}_*$, and

$$\begin{aligned} \frac{a^3}{P^2} &= \frac{G\mathcal{M}_*}{4\pi^2} \\ a^3 &= G\mathcal{M}_* \frac{P^2}{4\pi^2} \\ a^3 &= G\mathcal{M}_* \left(\frac{P}{2\pi}\right)^2 \\ a &= \left[G\mathcal{M}_* \left(\frac{P}{2\pi}\right)^2 \right]^{\frac{1}{3}} \end{aligned} \tag{15.1}$$

The mass of the star, \mathcal{M}_* , is generally known. The period, P , is the time between two successive transits. In practice, we want to see three transits in a row, to be sure that we’re looking at passes of the same planet (there might be others in its solar system). If the time between each pair of transits is the same, we’ve probably got the same planet in each case. In Equ. 15.1, \mathcal{M}_* must be in kilograms, and P is in seconds. This will give you a semi-major axis in meters.

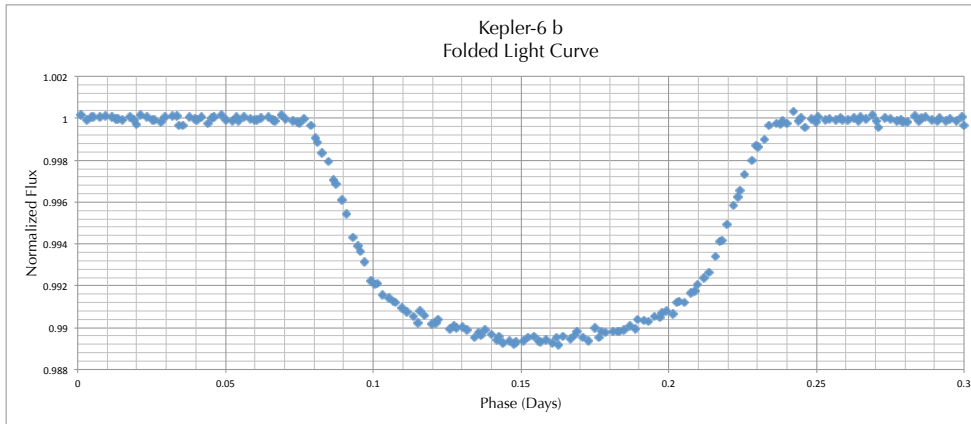


Figure 15.1: Folded light curve of Kepler-6b, showing the transit.

15.1.2 Measuring the Size of the Planet

Outside of a transit, we detect the total flux of the star, F_{tot} . During a transit, the planet blocks part of the flux of the star. If the star is evenly illuminated on its surface (not entirely true, but we'll start with this assumption), then the planet blocks some of the starlight, so we detect a lower flux, F_{trans} . The amount by which the total light is dimmed, $F_{\text{tot}} - F_{\text{trans}}$, is to the total light F_{tot} as the planet's area is to the star's area. [awk] That is,

$$\frac{F_{\text{tot}} - F_{\text{trans}}}{F_{\text{tot}}} = \frac{A_{\text{p}}}{A_{*}}.$$

Since both the star and the planet are circular, we can write their area in terms of radius, $A = \pi r^2$. Substituting this in and simplifying the left hand side,

$$1 - \frac{F_{\text{trans}}}{F_{\text{tot}}} = \frac{\pi R_{\text{p}}^2}{\pi R_{*}^2}$$

Canceling out the factors of π and solving for the radius of the planet, we obtain

$$\boxed{R_{\text{p}} = R_{*} \sqrt{1 - \frac{F_{\text{trans}}}{F_{\text{tot}}}}} \quad (15.2)$$

We measure F_{trans} and F_{tot} directly from the dip in the light curve, and we look up the radius of the star, R_{*} , from a catalog (See §15.2.2). If the

light curve is normalized (as in Fig. 15.1), then $F_{\text{tot}} = 1$, and the equation simplifies.

15.1.3 Calculating the Temperature of the Planet

Now that we know how far the planet is from its host star, and we know how big the planet is, we can estimate the planet's temperature. We'll apply the principle of "radiative balance," which states that when it is in thermal equilibrium and a constant temperature, a planet radiates out as much energy per time (power or luminosity) in the form of light as it absorbs from its star. We can write this as

$$L_{\text{em}} = L_{\text{abs}},$$

where L_{em} is the luminosity—the power (energy per second)—emitted from the planet by thermal radiation, and L_{abs} is the power absorbed from its host star.

Emission

Recall the Stefan-Boltzmann Law, which states that the flux (brightness) of light emitted from a blackbody depends on its temperature:

$$F_{\text{em}} = \sigma T_{\text{p}}^4,$$

where $\sigma = 5.6704 \times 10^{-8} \frac{\text{W}}{\text{m}^2\text{K}^4}$ is the Stefan-Boltzmann constant, and T_{p} is the blackbody's temperature (the planet, in this case). Flux is energy per time per *area*, which in this case is the surface area of the planet. Since flux is power per area, $F = \frac{L}{A}$, then the luminosity emitted is the flux times the surface area, giving us simply the energy per time:

$$L = FA.$$

The surface area of a sphere is $A = 4\pi R^2$, and so we get

$$L_{\text{em}} = \sigma T_{\text{p}}^4 4\pi R_{\text{p}}^2.$$

Absorption

The other side of the equation is the luminosity absorbed by the planet. Recalling that $L = FA$,

$$L_{\text{abs}} = (\text{absorption coefficient}) \times F_{\text{inc}} A_{\text{cross}},$$

where “absorption coefficient” accounts for how well the planet absorbs light, F_{inc} is the incident flux from the star hitting the planet, and $A_{\text{cross}} = \pi R_{\text{p}}^2$ is the cross-sectional area of the planet, a circle. Why do we use the entire surface area for emission but only the cross-section area for absorption? Because the starlight is only shining on one side of the planet! The planet “intercepts” a cone of starlight as wide as its cross-section.

So what is this “absorption coefficient”? Well, dark surfaces absorb (and emit) light better than light surfaces do, so we can account for this with a number that scales with how dark it is—how well it absorbs light. However, there’s already a coefficient called “albedo,” α , which is how *light* a surface is. Albedo is 0 for a perfectly black, absorbing surface and 1 for a perfectly white, reflecting surface. So it’s the opposite concept from our absorption coefficient, and we can write

$$\text{absorption coefficient} = 1 - \alpha.$$

We don’t know what the planet’s albedo really is without more information, but based on models, we’ll take it to be $\alpha = 0.07$.

The incident flux is the intensity of light from the star at the planet’s distance from it. As the light from the star travels out into space, it expands out evenly in all directions, spreading out in the shape of a sphere. When it reaches the planet, that sphere of light has a radius equal to the planet’s distance from the star, a . So recalling that $F = \frac{L}{A}$, we write

$$F_{\text{inc}} = \frac{L_*}{4\pi a^2},$$

where the surface area of the sphere of light is $4\pi a^2$. We can write the luminosity of the star, L_* , in terms of its temperature and size using the Stefan-Boltzmann equation and surface area, just like we did for the planet above:

$$L_* = \sigma T_*^4 4\pi R_*^2.$$

Substituting all of these components into our first equation, we find

$$L_{\text{abs}} = (1 - \alpha) \frac{\sigma T_*^4 4\pi R_*^2}{4\pi a^2} \pi R_{\text{p}}^2$$

Simplifying,

$$L_{\text{abs}} = (1 - \alpha) \sigma T_*^4 \frac{R_*^2}{a^2} \pi R_{\text{p}}^2$$

Equilibrium Temperature

Recall

$$L_{\text{em}} = L_{\text{abs}}$$

$$\sigma T_{\text{p}}^4 4\pi R_{\text{p}}^2 = (1 - \alpha) \sigma T_{*}^4 \frac{R_{*}^2}{a^2} \pi R_{\text{p}}^2$$

The σ and πR_{p}^2 cancel out on both sides.

$$T_{\text{p}}^4 = \frac{1}{4}(1 - \alpha)T_{*}^4 \frac{R_{*}^2}{a^2}$$

Solving for T_{p} ,

$$T_{\text{p}} = T_{*} \left[(1 - \alpha) \frac{R_{*}^2}{a^2} \right]^{\frac{1}{4}}, \quad (15.3)$$

where T_{*} is the star's surface temperature in degrees K, the albedo is $\alpha \sim 0.07$ (an assumption), R_{*} is the star's radius, and a is the planet's orbital semi-major axis. Both R_{*} and a must be in the *same units*, regardless of what units those are. That way, the units will cancel out. It's best to put them in either m or km, but be sure which you're using. All of the units on the right hand side will cancel out, except for degrees K, and so the planet's temperature, T_{p} , will be in K.

While we've cleaned up all of that mess in the derivation, this is still a somewhat obscure-looking equation. Let's take a look at it to see what's going on. On the right hand side, we have T_{*} , so the temperature of the planet is proportional to the temperature of the star. That makes sense. The hotter the star, the more light shines on the planet, heating it up. We have $1 - \alpha$, so the whiter the planet (greater albedo, α), the less light it absorbs, and $1 - \alpha$ lowers its temperature. There is R_{*} , so the bigger the star, the more light shines out, heating the planet. And finally $1/a$, so the farther away the planet is from its star (greater a), the less light reaches it, and the cooler it is. All of these check out with what we would expect.

15.2 Target List

The MAST archive at Space Telescope has a very clean set of light curves for five of the first Kepler planets found. For this exercise, then, we will analyze the planets listed in Table 15.1.

Table 15.1: Kepler Targets

Common Name	Kepler Catalog #
Kepler-4 b	KPLR11853905
Kepler-5 b	KPLR8191672
Kepler-6 b	KPLR10874614
Kepler-7 b	KPLR5780885
Kepler-8 b	KPLR6922244

15.2.1 Obtaining Planet Transits

Go to the MAST (Mikulski Archive for Space Telescopes) archive at archive.stsci.edu¹ to find the time series data on these five Kepler targets. From the MAST home page, select High-Level Science Products (HLSP). Scroll down and select Kepler Objects (KEPLER_HLSP). This page has both transit and radial velocity data for the first five planets found with the Kepler Space Telescope.

For each planet:

1. Download the “Kepler Time Series,” which records the transit data, and save it as a text file or CSV. This will be used to find the planet period and radius.
2. Download the “Ground-based Time Series,” which records radial velocity data, and save it as a text file or CSV. This will be used to find the planet mass. You will only need to keep the first two columns.

Important Note: In some browsers on Windows computers (often Internet Explorer, but check any of them for this bug), the file is saved without any carriage-returns at the end of the lines, so the entire file comes out as a single line. If this happens, first try opening the file in a different text editor. If that does not fix the problem, switch to a different browser.

15.2.2 Obtaining Stellar Data

Go to this website to look up the host star data for each of these planets: exoplanet.eu

¹If you can’t access the website in your browser, try using `https` rather than `http`.

For each planet:

1. Select **All Catalogs**.
2. In the **Planet Search** field, search for planet Kepler 6b (for example) with this syntax: ‘‘**kepler-6 b**’’ **IN name**
Include the quotation marks. **Important Note:** The website is very picky with the planet name, and while capitalization does not matter, you must use the correct spacing and punctuation. Kepler 6b, for example, must be written **kepler-6 b**. The other Kepler planets will have the same naming format.
3. Select the result that comes up.
4. Look on the right side of the page for **Star Data**. Record the star’s mass (in solar masses, \mathcal{M}_{Sun} or \mathcal{M}_{\odot}) and radius (in solar radii, R_{Sun} or R_{\odot}). You will convert these to the units you need in your final calculations.

15.3 Analyzing a Light Curve With a Spreadsheet

[See video.]

15.4 Getting the Planet’s Mass From Radial Velocity

Get the radial velocity (Doppler shift) table for the star and import it into Excel. If it’s short enough, it may be easier to copy and paste each individual number into a cell, but be sure to copy the negative signs when they appear. The columns we are interested in are the time the observation was made (“Barycentric Julian Date,” in days) and the radial velocity in m/s.

Using your results from the transit plots, apply the modulo function to the Julian Date as you did before, using the same orbital period you found in your earlier work, and call this the “Modulo Julian Date.” Now plot the radial velocity *vs.* modulo Julian Date, and you’ll see a graph resembling a very sparsely-sampled sine wave. This shows how fast the star is moving towards

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and away from us as the planet orbits it. We can use this to determine the planet's mass.

Identify the fastest speed the star moves, whether it's positive or negative. Take the absolute value and call that $v_{*, \max}$.

Then the mass of the planet, \mathcal{M}_p , is

$$\mathcal{M}_p = \frac{v_{*, \max}}{\sin i} \sqrt{\frac{a \mathcal{M}_*}{G}},$$

where i is the inclination angle of the planet's orbit, a is its semimajor axis (distance from the star to the planet), \mathcal{M}_* is the mass of the star, and G is Newton's gravitational constant. The inclination angle is the angle between the axis of the planet's orbit and our line of sight. Since the planet crosses in front of the star as we look at it, that is going to be very, very close to 90° . We can actually calculate it more exactly from our transit data, but for our purposes, 90° is close enough. Since $\sin 90^\circ = 1$, then we are simply left with

$$\boxed{\mathcal{M}_p = v_{*, \max} \sqrt{\frac{a \mathcal{M}_*}{G}}.} \quad (15.4)$$

For the units to work correctly, remember that masses should be in kg, distances in m, and speeds in m/s.

The density of the planet is found by dividing its mass by its volume. Since it is a sphere, the volume is $V = \frac{4}{3}\pi r_p^3$, where r_p is the planet's radius, and the density, ρ_p (in kg/m^3), is

$$\rho_p = \frac{\mathcal{M}_p}{(4/3)\pi r_p^3}.$$

It is often convenient to convert the density into grams per cubic centimeter, because the gram was originally defined as the mass of one cubic centimeter of water. The unit conversion is simple—just divide kg/m^3 by 1,000: $\rho(\text{g}/\text{cm}^3) = \frac{\rho(\text{kg}/\text{m}^3)}{1000}$, and note for comparison that $\rho_{\text{water}} = 1 \text{ g}/\text{cm}^3$.

The density gives us a clue what the planet is likely to be made of. Different kinds of materials have different densities, but they may not be the same density that substance would have sitting in a lab on Earth. The exact density depends on the strength of the gravitational force squeezing them...which itself depends on the density of the material making up the planet! Fortunately, planetary geologists have come up with charts showing

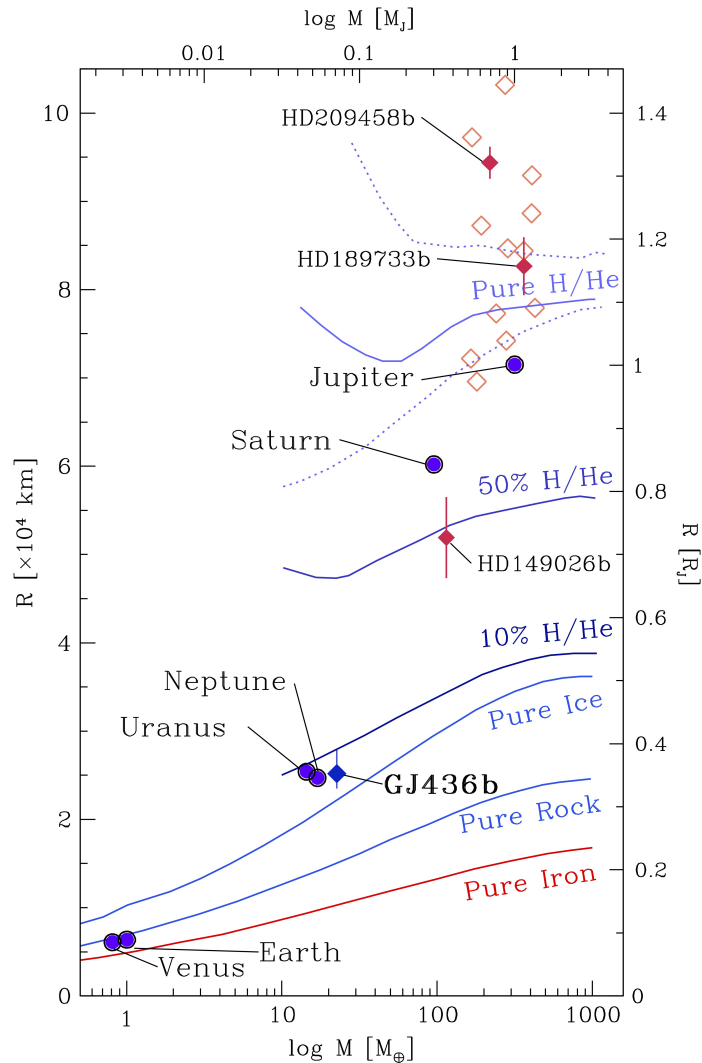


Fig. 9. Planetary mass–radius diagram comparing the position of Solar System planets, transiting hot Jupiters (diamonds), and the Neptune-like GJ 436 b. The lines indicate the position of the Fortney et al. (2007) models for different compositions: pure iron, pure silicate, pure water ice (with thermal profiles from Solar System planets), and models for irradiated planets at 0.1 AU from a Solar-type star with a fraction of 10%, 50% and 100% of Hydrogen/Helium. The dotted lines show the models for a cold ($a = 10 \text{ AU}$) and very hot ($a = 0.02 \text{ AU}$) pure H/He gas giant. From Gillon (2007).

Figure 15.2: Planet composition as a function of size (radius, R) and mass (M). The horizontal scales give mass in Earth masses (M_{\oplus} , bottom) and Jupiter masses (M_J , top). The vertical scales give radius in 10,000s of km ($R [\times 10^4 \text{ km}]$) and Jupiter radii (R_J). The curved lines show the size as a function of mass for planets of different compositions, and density increases down and to the right. We can look at the location of the Earth to see how to read this. It lies between the pure rock and pure iron curves, but close to the pure rock, so we would conclude that Earth is mostly rock, with some component of iron (the core).

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the densities of different mixtures of gasses, ices, and rock for planets of a given size and mass (Figure 15.2).