Chapter 21

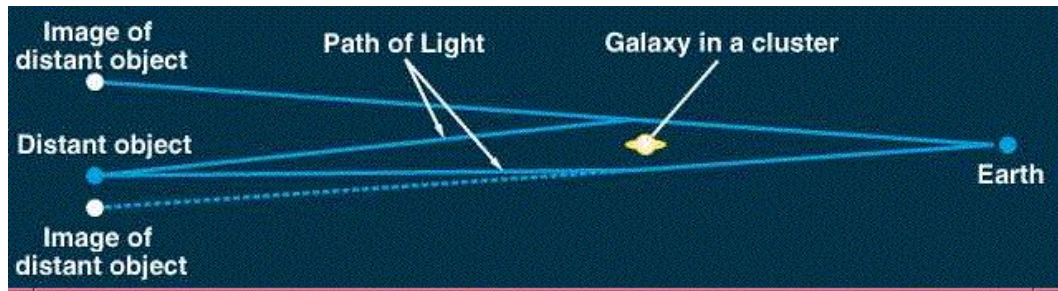# Measuring dark matter from gravitational lensing

Figure 21.1: Schematic of strong gravitational lensing, showing light paths from source, past lens, to observer. (c) UCSD

21.1   Gravitational Lensing

Gravitational lensing occurs when the light from a distant *source* (left side of figure), such as a quasar or galaxy, is deflected by the gravitational pull of another galaxy or galaxy cluster (the *lens*, in the middle of the figure) before reaching the *observer* (Fig. 21.1). In this 2-dimensional schematic, we see two *images* of the source, angling above and below the lens. But there are light rays angling out of the page or down into it, and these are deflected, too. So in principle, we might have several images of the source arrayed around the lens, two for each axis of symmetry in the lensing galaxy. If the lens is elliptical-shaped, and the source is well-aligned, we can get four images (Fig. 21.2, top), called an *Einstein cross*. Across the narrow, or *minor* axis of the lens, the lens' mass is packed in more densely per angle, and this splits the images wider. Across the longer, or *major* axis, the lens' mass is more spread out on the sky, and this splits the images less. The effect is easily seen in the right-hand image of the top of Fig. 21.2.

In a few cases, the lens and source are so well aligned, and the lens is so symmetrical, that there is an infinite number of images circling the lens, forming a so-called *Einstein ring* (Fig. 21.2, bottom). The Einstein ring occurs at a very specific radius, the Einstein radius, which we'll mark as θ_E: θ for the angular distance on the sky and E for Einstein. Let's imagine a lens is far from the line of sight of the source, so that the source's image is not distorted. Let's move the lens closer to the line of sight of the source. As we do so, we notice the primary image is distorted more and more, becoming magnified and stretched out into an arc that gets closer to the imaginary circle

Gravitational Lenses
HST · WFPC2
PRC95-43 · ST ScI OPO · October 18, 1995 · K. Ratnatunga (JHU), NASA

J073728.45+321618.5   J095629.77+510006.6   J120540.43+491029.3   J125028.25+052349.0

J140228.21+632133.5   J162746.44-005357.5   J163028.15+452036.2   J232120.93-093910.2

Einstein Ring Gravitational Lenses
Hubble Space Telescope · Advanced Camera for Surveys

NASA, ESA, A. Bolton (Harvard-Smithsonian CfA), and the SLACS Team   STScI-PRC05-32
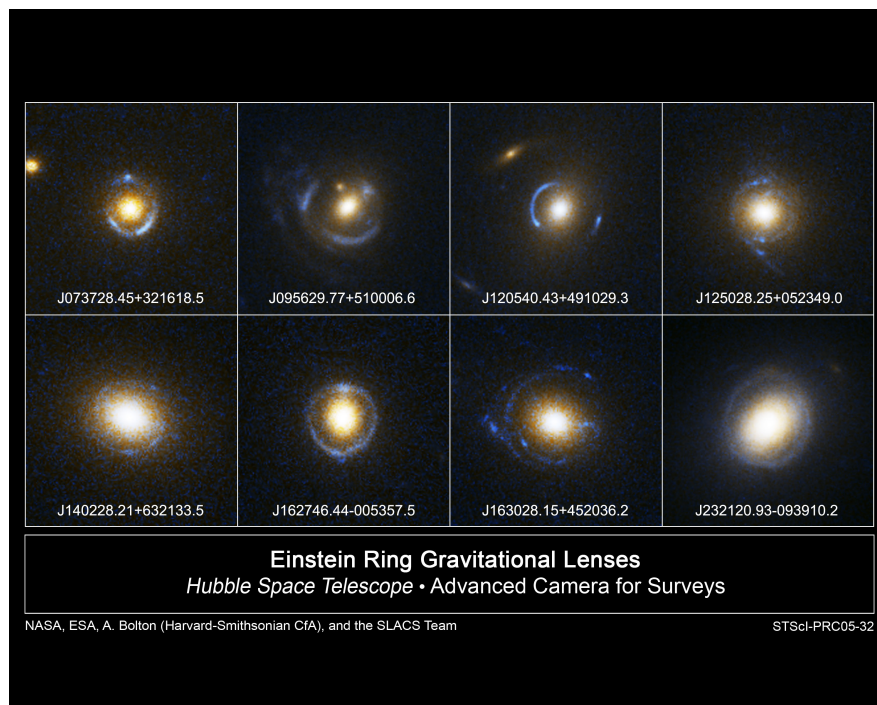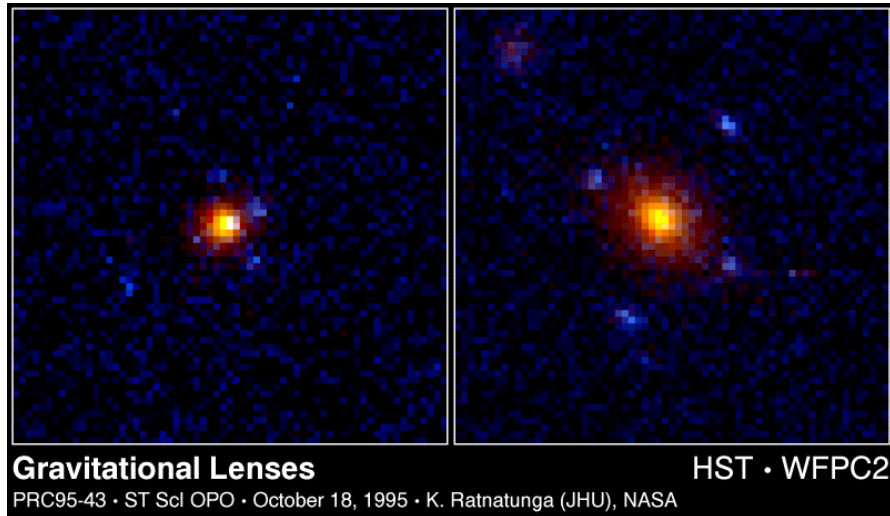
Figure 21.2: **Top:** Einstein crosses. **Bottom:** Einstein rings encircling the lensing galaxy. The *lenses* are the central galaxies, often colored red to yellow because they are usually ellipticals with older stellar populations. The lensed *images* are arranged around the lens and are often (but not always) blue, either because they are quasars or star-forming spiral galaxies in the early universe.

defining the Einstein radius. At the same time, we see a smaller image inside the Einstein radius and on the opposite side of the lens. It moves away from the lens center, getting bigger, brighter, and closer to the Einstein radius. When the lens is perfectly lined up with the source, the two images meet and merge into an Einstein ring—one image from inside the radius and one from outside. You can try out a demonstration of this effect online at `leo.astronomy.cz/grlens/grl9.html`.

21.1.1   Deriving the Total Mass

Approximate Form

This part is useful for doing a "sanity check" on your math, but Observational Astronomy students should use the correct form, below.

For the simple case of a compact, symmetrical lens, the Einstein radius can be approximately derived in terms of just the mass of the lens, $\mathcal{M}$, and the distance to the lens, D. In reality, it depends on the distance to both the lens and the source, but if all of the distances are of the same order of magnitude, we can simplify. The Einstein radius is then approximately (Padmanabhan, p. 204, equ. 3.200)

$$\theta_{\mathrm{E}} \approx 2'' \left(\frac{\mathcal{M}}{10^{12}\mathcal{M}_{\odot}} \right)^{1/2} \left(\frac{D}{3,000 \text{ Mpc}} \right)^{-1/2},$$

where θ_{E} is the angular radius of the ring, $''$ refers to arcseconds, $\mathcal{M}$ is the mass of the lensing galaxy (note that we have divided this by one trillion solar masses), and D is the distance to the lens (which we have divided by 3,000 Mpc). This means that we can predict the size of the ring, if we know the mass of the lensing galaxy and its distance.

In our case, though, let's say we've already measured the radius of the ring, θ_{E}, and we want to estimate the mass. So solve the equation for the mass:

$$\mathcal{M} \approx 10^{12}\mathcal{M}_{\odot} \left(\frac{\theta_{\mathrm{E}}}{2''} \right)^{2} \left(\frac{D}{3,000 \text{ Mpc}} \right).$$

As a simple example, if we have a lens at a distance of $D = 3,000$ Mpc, with an Einstein ring at a radius of $\theta_{\mathrm{E}} = 2''$, then we will find its mass to be one trillion solar masses.

Correct Form

Observational Astronomy students should use the equation in this section

$$\theta_{\mathrm{E}} \equiv \sqrt{\frac{4\mathcal{M}G}{c^2} \frac{D_{\mathrm{SL}}}{D_{\mathrm{SO}} \, D_{\mathrm{LO}}}}, \tag{21.1}$$

where the D's are all *angular diameter distances*. D_{SL} is the distance between the *Source* and the *Lens*, D_{SO} is the distance between the *Source* and the *Observer*, and D_{LO} is the distance between the *Lens* and the *Observer*.

Solving for the mass, we obtain

$$\mathcal{M} = \frac{c^2}{4G} \frac{D_{\mathrm{SO}} \, D_{\mathrm{LO}}}{D_{\mathrm{SL}}} \, \theta_{\mathrm{E}}^2. \tag{21.2}$$

Substituting in the values for c and G, we wind up with an equation that wants us to give it the angle in radians, distances in meters, and it will give us the mass in kg, but that's *very* inconvenient for the tiny angles and the big distances and masses we'll have in astronomy. Our angles will be around one millionth of a radian, our distances will be thousands of Megaparsecs, and our masses will be trillions of solar masses. So after converting the units and dividing by convenient factors, we come up with a version that keeps every term close to unity:

$$\boxed{\left(\frac{\mathcal{M}_{\mathrm{total}}}{10^{12} \mathcal{M}_{\odot}} \right) = 1.47 \left(\frac{\theta_{\mathrm{E}}}{2''} \right)^2 \left(\frac{D_{\mathrm{SO}} \, D_{\mathrm{LO}}}{3{,}000 \text{ Mpc} \times D_{\mathrm{SL}}} \right)} \tag{21.3}$$

I'll give you some practical tips in solving this equation for the mass. Look at how the units on the right hand side will all cancel. The Einstein radius, θ_{E} will usually be close to $1''$, and the arcseconds will cancel out, giving you something close to $\left(\frac{1}{2} \right)^2$, a nice, simple, dimensionless number. Your distances will generally be a few times 1,000 Mpc. You don't actually need to multiply $3{,}000 \text{ Mpc} \times D_{\mathrm{SL}}$. Instead, divide one of the distances in the numerator by 3,000 Mpc, and divide the other one by D_{SL}! This means you'll simply have a couple of numbers close to unity to multiply together in that last term. This leaves the right hand side with a pure, dimensionless number, probably close to unity. That is how many trillions of solar masses the galaxy weighs. You can multiply both sides by $10^{12} \mathcal{M}_{\odot}$ (one trillion solar masses) to make that clear.

21.1.2 Finding Angular Diameter Distances

We have three distances involved in this problem: D_{SL}, D_{LO}, and D_{SO} (Fig. 21.3), where S stands for *Source*, L for *Lens*, and O for *Observer*. The distances we use here, though, are not the distances you would get if you stretched out a tape measure over those millions of parsecs. Rather, they
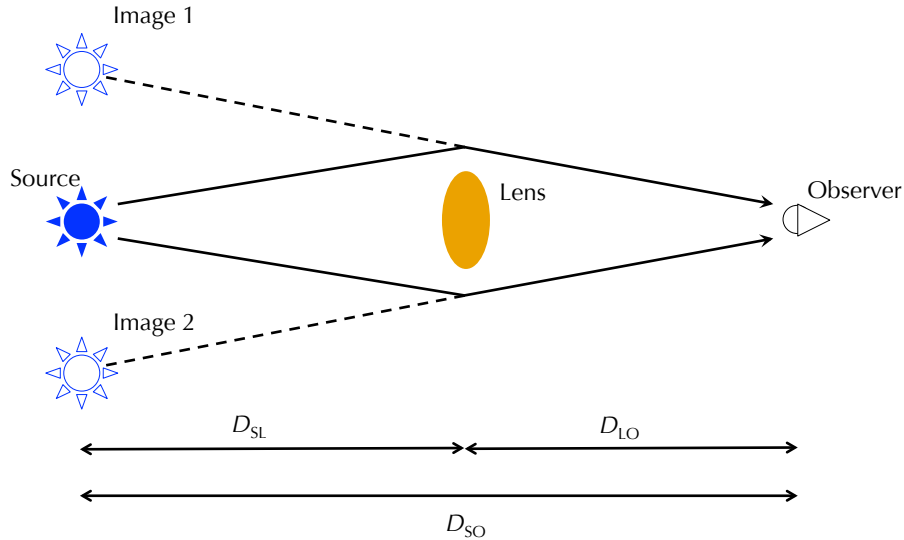
Figure 21.3: Side view of positions and distances in a gravitational lens.

are what is called *angular diameter distance*, which is the ratio of a galaxy's physical diameter to its angular diameter. For nearby galaxies, the angular diameter distance is about the same as its physical distance, but because of the expansion of the universe, this doesn't hold true for more distant galaxies. In fact, once they get past a redshift of $z \sim 1$, galaxies don't look smaller with greater distance, but *larger*! So angular diameter "distances" are peculiar things. They don't add the way you'd expect, either. Despite the way it looks in Fig. 21.3, $D_{\mathrm{SO}} \neq D_{\mathrm{SL}} + D_{\mathrm{LO}}$.

Given a galaxy's redshift, its angular diameter distance can be calculated, but it requires doing an integral. Fortunately, you don't have to do this yourself. Ned Wright has an online calculator that we'll use to get both the angular diameter distance (there llsted as "angular size distance," D_{A}) and the "comoving radial distance," which we'll label as χ:
`www.astro.ucla.edu/~wright/CosmoCalc.html`
Go to Ned Wright's calculator and enter the redshift, z, for the distance you want to calculate. Leave everything else in the default values[1]. Now

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[1]Hubble's constant should be $H_0 = 69.6$, the density of matter in the universe should

press the `General` button, which calculates the distances. Write down the comoving radial distance in units of Mpc, the angular size distance in Mpc, and the luminosity distance in Mpc.[2]

The distance D_{SO} is the angular diameter distance at the redshift of the source. D_{LO} is the angular diameter distance at the redshift of the lens. For D_{SL}, we calculate it from the comoving radial distances and the source redshift:

$$D_{\mathrm{SL}} = \frac{\chi_{\mathrm{S}} - \chi_{\mathrm{L}}}{1 + z_{\mathrm{S}}}, \tag{21.4}$$

where χ_{S} and χ_{L} are the comoving radial distances of the *Source* and the *Lens*, respectively, and z_{S} is the redshift of the *Source*.

21.1.3 Measuring θ_{E}

How do we measure the radius of the ring? Open the image in DS9 and click to create a circular "region" on the lensing galaxy. Select the region and drag the corners to scale the size until it matches the Einstein ring in the picture (make sure it's concentric with the ring). Then double-click on the region or select `Region`→`Get Information` to bring up the details. Beside `Radius`, make sure `WCS` (World Coordinate System) and `arcsec` are selected, and write down the radius in arcseconds. Using J0737+3216 as an example, we see in Fig. 21.4 that $\theta_{\mathrm{E}} = 1.05''$.

21.1.4 Measuring Stellar Mass

In the previous sections, we calculated the total mass of the galaxy within the Einstein ring, which includes both normal matter (stars, gas, and dust) and dark matter. Now we'll calculate the amount of normal matter, using the amount of light emitted by the galaxy within the Einstein ring.

Measure the Amount of Starlight

Open the gravitational lens image within APT and select the $\boxed{\texttt{More Settings}}$ button. The Hubble Space Telescope images we're using have already had their sky levels subtracted, so we will choose `Sky Algorithm A`, which doesn't

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be Omega$_{\mathrm{M}} = 0.286$, and the vacuum energy density should be Omega$_{\mathrm{vac}} = 0.714$.

[2]We will need the luminosity distance for the lens itself when we calculate its absolute magnitude in §21.1.4.
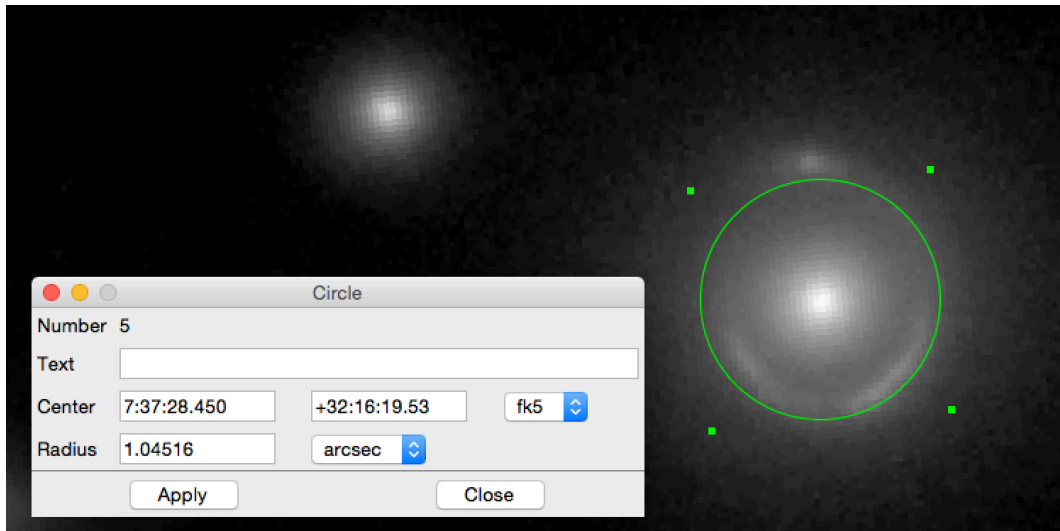
Figure 21.4: Using DS9's region tool to measure the Einstein radius of J0737+3216. Note that the circular region is concentric with the lensing galaxy but not with the apparent shape of the ring itself. This does not look like a perfectly-circular ring to me, so I have adjusted the circle so that it misses the inner parts of the ring by about the same amount that it misses the outer parts of the ring.

Table 21.1: Filter Zeropoints

Telescope	Camera	Setting	Filter	Zeropoint
HST	ACS	WFC	F555W	25.79
HST	ACS	WFC	F814W	25.53

Telescope: HST—Hubble Space Telescope. Camera: ACS—Advanced Camera for Surveys. Setting: WFC—Wide-Field Camera. Filter: F555W (green), F814W (infrared).

subtract the sky. Enter the `Magnitude zero point` for the camera and filter you are using from the Table 21.1. Select $\boxed{\texttt{Apply Settings}}$ and $\boxed{\texttt{Close Window}}$.

Now we need to adjust the image display so that we can find the gravitational lens and place the aperture. It is helpful to open the image in DS9 first and note the dimmest and brightest pixel values in the lens area. In APT, enter those same values in the fields for `Lower bound of stretch` and `Upper bound of stretch`, respectively. Hit `Enter` on your keyboard after you type the last of these. Next, toggle through the color-scale display options with the $\boxed{\texttt{Stretch-type toggle}}$ button until it displays `Logarithmic stretch` above it. The options in my APT version are `Linear`, `Logarithmic`, and `Histogram Equalization`. `Linear` makes a pixel that has twice the value of another be twice as bright. This is fine if you don't have much dynamic range, but if there are very bright areas and very dark areas, you won't be able to see the details in both at the same time. `Logarithmic` is what I prefer. It mimics the way our eyes respond to light, decreasing the differences among very bright pixels and displaying a much wider dynamic range in the image. Next, adjust the `Dynamic range` slider the right, until you are able to see the details you want in the image.

Now we will look for the gravitational lens in the image and place the aperture on it. If you have a large image, you can't see very much of it in the main viewing window, so select the $\boxed{\texttt{Thumbnail}}$ button to display the entire image (you can close it as soon as you see where it is). Locate your lens and use the sliders on the bottom and right sides of the main viewing window to bring it into view there. Click on the center of the lensing galaxy to place the aperture there. It will probably snap into position in the very center, if

you get close.[3] You may find it useful to magnify the image with the buttons below the viewing window. If the lens disappears from view when you do so (this usually resets the viewing window to another part of the image), just select $\boxed{\texttt{Find Aperture}}$, and it will be centered.

Now we will adjust the aperture shape and size. Select the $\boxed{\texttt{Big Button on the Right}}$, which opens the `Elliptical-Aperture Attributes` window. If the lensed galaxy forms an Einstein ring, or close to it, or if its components are about equally distant from the center, then we'll use a circular aperture, which I'll describe first. If it has a fairly elliptical arrangement, we'll use an elliptical aperture, and I'll describe it second. For a circular aperture, make sure the `circular aperture` box is checked, and adjust the `Major Radius` until the aperture circle lies *just inside* the inner edge of the Einstein ring. Note that you can either use the slider or enter a number directly to adjust it. We do not want to include any light from the ring itself in the aperture. If the ring is slightly off-center from the lensing galaxy, then you can shift its position with the `Center (X, Y)` fields at the top. If the lensed image is noticeably elliptical, or if it has several components that are arranged at different distances from the galaxy center, then de-select the `circular aperture` box and adjust the `Major radius` and `Minor radius` separately. Adjust the `Rotation angle` until it matches the shape of the lensed image. Remember to keep the aperture just *inside* the edge of the lensed image, because we don't want any of its light in our measurement. When you are finished, select $\boxed{\texttt{Apply}}$ and close the window.

Now we're ready to compute the lensing galaxy's magnitude. Select $\boxed{\texttt{Recompute Photometry}}$ and record the `Magnitude` displayed in blue near the bottom of the screen. This is the apparent magnitude of the lensing galaxy within the Einstein ring.

Convert apparent magnitude to absolute magnitude

To convert this apparent magnitude into an absolute magnitude, we will use Equ. 5.4:

$$\boxed{M = m - 5\log\left(\frac{D_{\mathrm{L}}}{\mathrm{Mpc}}\right) - 25 - A - K},$$

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[3]This is called the "centroid" algorithm, and it's a very handy tool used in other astronomical image programs.

where D_{L} is the *luminosity distance* to the galaxy (which I had you record above), A is the Galactic extinction for this filter and direction in the sky, and K is the K-correction for this galaxy type in this filter. Be sure to review §5.3.4, which explains the concepts. A table of K-corrections for different filters, galaxy types, and redshifts can be found in Table 5.1.

Convert absolute magnitude to stellar mass

Now we use this absolute magnitude to find the mass of stars in the lensing galaxy, within the region of the Einstein ring. For this, we have to make use of a *mass-to-light ratio*, $\mathcal{M}/L$. I've taken that ratio and converted it to magnitudes

$$\boxed{\mathcal{M}_* = 1.23 \times 10^{-0.4(M_{F814W}-5.19)}\mathcal{M}_\odot,} \tag{21.5}$$

where M_{F814W} is the galaxy's absolute magnitude in the Hubble WFPC2 camera's $F814W$ filter (which you found in the equation above in §5.4, cited above), and $\mathcal{M}_*$ is the mass of stars in the galaxy, in units of solar masses $(\mathcal{M}_\odot)$.[4] I have specified in Equ. 21.5 that we're using the $F814W$ filter magnitude in the equation, because while Equ. 5.4 and most other equations work with any filter, the specific numbers in Equ. 21.5 are only valid for the $F814W$ filter.

21.1.5 Target List of Gravitational Lenses with Einstein Rings

Table 21.2 contains a list of gravitational lenses to be observed. Their RA and Dec are listed for reference in looking up their Galactic extinction or for other information from outside sources. The redshift of both the lens (z_{L}) and source (z_{S}) are given. Both are needed for calculating the total mass. Only the lens redshift is used in the stellar mass calculation.

21.1.6 Finding the Dark Matter

So you have found both the *total mass* from the radius of the Einstein ring and the *stellar mass* from the lens magnitude. The total mass is the sum of

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[4]Zibetti (2002) gives $\mathcal{M}/L_H = 1.23$, where L_H is the H-band galaxy luminosity. I have applied elliptical galaxy colors from Mannucci (2001) to apply this to the $F814W$ filter (I-band) and converted it to magnitudes.

Table 21.2: Gravitational Lens List

#	Short Name	RA (hh mm ss.sss)	Dec (dd mm ss.sss)	z_L	z_S
1.	J0216−0813	02 16 52.530	−08 13 45.55	0.332	0.523
2.	J0737+3216	07 37 28.434	+32 16 18.60	0.322	0.581
3.	J0912+0029	09 12 05.31	+00 29 01.2	0.164	0.324
4.	J0959+0410	09 59 44.070	+04 10 17.54	0.126	0.535
5.	J1250+0523	12 50 28.251	+05 23 49.05	0.232	0.795
6.	J1402+6321	14 02 28.028	+63 21 33.71	0.205	0.481
7.	J1627−0053	16 27 46.437	−00 53 57.62	0.208	0.524
8.	J1630+4520	16 30 28.222	+45 20 36.35	0.248	0.793

Columns: (1)—Simplified name; (2) & (3)—Equatorial coordinates (hours minutes seconds); (4)—Lens redshift; (5)—Source redshift. The names are not recognized galaxy names; they are only shortened forms of their coordinates. If looking them up in a catalog or paper, you should use the coordinates and redshift. These images are taken with the Wide Field Planetary Camera 2 (WFPC2) on board the Hubble Space Telescope, using the $F814W$ filter, which is similar to the I_C (Cousins I), a standard near-infrared filter.

the stellar matter and the dark matter. So to find the mass of dark matter, subtract the stellar mass from the total.

The most interesting thing about the mass of dark matter is what fraction of the total it makes up. Studies have shown it usually to be over 90% of the total mass of a galaxy. Simply divide the mass of dark matter by the total mass, and you have the dark matter fraction of this lens.

21.1.7 Requirements for the Lab Report

For the lab report, you must follow the standard format, being sure to include the following items:

1. A close-in image of each gravitational lens system, showing the lens and the Einstein ring.

2. Calculations of

   - total mass
   - stellar mass
   - mass of dark matter
   - dark matter as a fraction of total mass

The results of the calculations should be listed in a single table.