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The Spatial Distribution of Pulsars and the Spiral Structure of the Galaxy

R. Ramachandran & A. A. Deshpande *Raman Research Institute, Bangalore 560 080*

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Abstract. We have looked for and found a possible spatial correlation between the present pulsar distribution and the estimated locations of the spiral arms at earlier epochs. Through a detailed statistical analysis we find a significant correlation between the present distribution of pulsars and the mass distribution (in the spiral arms) expected about 60 Myr ago for a corotation resonance radius of 14kpc. We discuss the implications of this correlation for the minimum mass of the progenitors of pulsars. Interpreting the spread in the locations of pulsars with respect to the past locations of the spiral arms as predominantly due to their space velocities, we derive an average velocity for the pulsar population.

Key words: Pulsars: velocities—galaxy: structure, dynamics—stars: end states.

Introduction

Ever since their discovery, pulsars have been the subject of many statistical studies. However, the importance of the connection between their birth places and the locations of spiral arms was not emphasized until the work of Blaauw (1985). From a detailed consideration Blaauw argued that the local birthrate of pulsars should equal the local deathrate of massive stars. He found that the regions of active star formation, the OB associations, contribute only a small fraction (~ 15%) of the local birthrate of pulsars; most of the pulsars come from the field population of 6–10$M_\odot$ stars. Hence, one of his conclusions was that *pulsars are*, *on the Galactic scale*, *tracers of regions of past spiral structure* (*20–50 Myr ago*) *rather than of 'active' spiral structure.* If this is true, one should find a spatial correlation between the global distribution of pulsars and the location of spiral arms in the past. Such a correlation may be reduced considerably if their progenitors and later pulsars move from their birth places by distances comparable to the interarm spacing. But if a correlation is found despite these effects, then it can provide a significant constraint on the velocity distribution of pulsars. In this paper, we have attempted to look for a possible spatial correlation of the 'global' pulsar distribution with the expected locations of the spiral arms at different epochs. A recent model of electron density distribution due to Taylor & Cordes (1993) is used to obtain more reliable distance estimates to pulsars from their dispersion measures. Fig. 1 shows the 'observed' pulsar distribution based on this model superimposed on the model electron density distribution. The true dis-

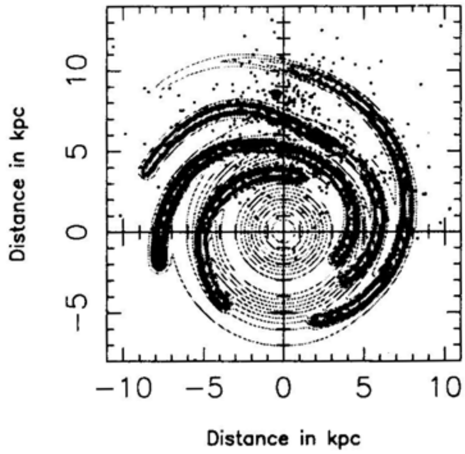

Figure 1. The new electron density distribution model of Taylor & Cordes (1993). Projected locations of all known pulsars are plotted as dots. Sun is at (0,8.5).

tribution can be obtained from this after accounting for the selection effects. Section 2 describes the method we have adopted to estimate the selection factors *as a function of position in the Galaxy.* The method of evaluating the correlation between the Galactic distribution of pulsars and the spiral pattern is discussed in section 3. We use a value of 14 kpc for the Galactic *corotation resonance radius* (following Burton 1971); this is needed to define the angular velocity of the spiral pattern. We argue that the present Galactic pulsar distribution of pulsars can be well correlated with the mass distribution in the spiral pattern about 60 Myr ago. In this section we also discuss the implications of this result for the minimum mass of pulsar progenitors. Our analysis shows that the minimum mass of the progenitors of pulsars may be as low as 7 $M_{\odot}$.

In section 4 we examine the *spread* in the locations of pulsars with respect to the past locations of the spiral arms and discuss possible explanations for it. By attributing this spread primarily to pulsar velocities and after allowing for the distance uncertainties, we obtain an estimate for the average velocity of pulsars.

2. Selection effects

The observed distribution of pulsars shown in Fig. 1 is expected to deviate systematically from the real distribution due to various selection effects which make detection of radio pulsars difficult. The detectability is affected by both the luminosity of the pulsar and the pulsed nature of the emission. The interstellar scattering and dispersion result in the broadening of the observed pulses reducing the peak pulsed flux. The effect of such factors can be readily seen in Fig. 2, where the distribution of minimum distances (d_{min}) from the nearest arm to the pulsars is plotted. The value of d_{min} is taken to be positive when it is from that side of the arm which is nearer to the sun and as negative otherwise. One would expect this distribution to be roughly symmetric if the probability of detection were uniform over the Galaxy. In reality, the probability of detection reduces with increasing distance from the Sun. Hence, pulsars closer to the Sun (which are assigned + ve d_{min} values) tend to skew the distribution of minimum

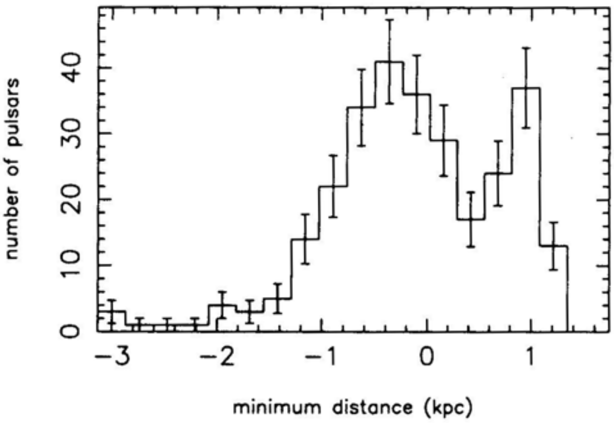

Figure 2. Distribution of minimum distances ($d_{\min}$) of pulsars to the nearest spiral arms. The value of $d_{\min}$ is taken as positive when it is from that side of an arm which is nearer to the sun and as negative otherwise. The peak at 0.9 kpc is identified to be due to the pulsars in the solar neighbourhood. Error bars indicate 1σ deviation on either side.

distances. The feature at $d_{\min} \approx 0{\cdot}9$ kpc can be attributed to the bias caused by the pulsars close to the Sun. The asymmetry seen in the observed distribution of $d_{\min}$ clearly stresses the need for compensation of selection effects. We give below the procedure we have adopted to compute the scale factors $S(R, z, \phi)$ (ratio of the total number to the number observed) to compensate for the selection effects as a function of the position in the Galaxy.

In the discussion to follow, our sample of pulsars is restricted only to those which in principle could have been detected by any one of eight of the major pulsar surveys namely, (1) Jodrell, (2) U. Mass–Arecibo, (3) Second Molonglo, (4) U. Mass–NRAO, (5) Princeton–NRAO phase 1, (6) Princeton–NRAO phase 2, (7) Princeton–Arecibo and (8) Jodrell-1400 MHz. Of the 570 pulsars known to data, only 325 pulsars satisfy the above mentioned selection criterion. The globular cluster pulsars, extragalactic pulsars and the millisecond pulsars are not included in this analysis.

2.1 *Scale Factors as a Function of the Position in the Galaxy*

A complete treatment of modelling various selection effects is given in Narayan (1987) (see also Bhattacharya *et al.* 1992 and Lorimer *et al.* 1993). The parameters which go into the determination of selection effects are (1) regions covered by the pulsar surveys, (2) their sampling rates and sensitivities, (3) scattering and dispersion smearing of the pulse profiles, and (4) the luminosity function of pulsars. The procedure we have used to determine the scale factors as a function of position in the Galaxy may be described as follows.

First, the scale factor as a function of pulsar period and luminosity $S(P, L)$ are estimated as (Narayan 1987),

$$S(P, L) \equiv \frac{\iint \rho_R(R)\rho_z(z) R\,\mathrm{d}R\,\mathrm{d}\phi\,\mathrm{d}z}{\iint_{\mathrm{obs}} \rho_R(R)\,\rho_z(z) R\,\mathrm{d}R\,\mathrm{d}\phi\,\mathrm{d}z} \tag{1}$$

here R is the galactocentric radius, z is the distance from the Galactic plane, P is the rotation period of the pulsar and $\dot{P}$ is its time derivative and L is the luminosity of the pulsar. The function ρ describes the distribution of pulsars with respect to the corresponding variable in the bracket. Here, the integral in the numerator is over the whole Galaxy and the integral in the denominator is over only those parts of the Galaxy where a pulsar of period P and luminosity L can be detected by at least one of the eight surveys included in our analysis. We have computed this scale factor as a function of P and L by doing a Monte Carlo simulation. A detailed description of the estimation of such scale factors can be found in Narayan (1987). For the computation of dispersion and scattering smearing we used the new electron density distribution model of Taylor & Cordes (1993).

A luminosity relation as a function of P and B then enables one to evaluate $S\,(P, B)$ from $S\,(P, L)$ where B is the magnetic field of the pulsar (see Narayan & Ostriker 1990; Prozynski & Przybycien, 1984 for the luminosity relation). We also take into account the dispersion of luminosity around the model luminosity as suggested by Narayan & Ostriker (1990).

Since the scale factor is the ratio of the *'true'* number of pulsars to the number of known pulsars, the true number of pulsars in a bin of width ΔP around the period P and ΔB around the field B can be given by

$$N_{\rm real}(P, B) = \sum_{i=1}^{n_{\rm psr}} S_i(P, B) \tag{2}$$

where $n_{\rm psr}$ is the number of known pulsars in that bin. We have used the usual relation $B^2 \propto P\,\dot{P}$. Although we have defined the number distribution as a function of (P, B),one could also define it as a function of $(P, \dot{P})$ (see Srinivasan 1991; Deshpande *et al.* 1994). Fig. 3 shows the expected distribution of the 'true' number of pulsars as a function of P and B, which we use to compute the scale factor $S(R, \phi, z)$ as a function of the location in the Galaxy (where R, ϕ and z are galactocentric radius, azimuth

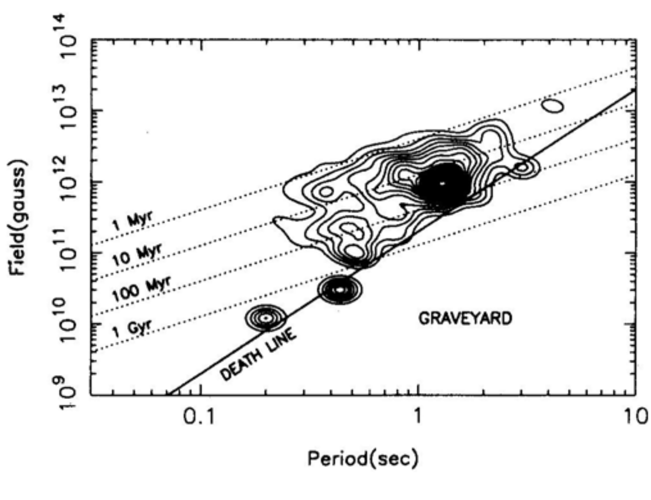

Figure 3. A contour representation of the distribution of 'true' number of pulsars as a function of pulsar period and magnetic field.The constant characteristic age lines corresponding to 1 Myr, l0 Myr, 100 Myr,and 1 Gyr are also plotted.(From Deshpande *et al.* 1994. Reproduced by permission).

angle and the height from the plane, respectively) in the Galaxy. Given the location of a pulsar in the Galaxy (from the selected sample) we populate that location with pulsars of various periods and magnetic fields with weightage given by equation (2). Then the ratio of the total number of pulsars projected to be at that location and the number of pulsars detectable by at least one of the eight surveys used in our analysis can be taken as the *scale factor corresponding to that location.* In our case, because we are primarily interested in the projected distribution in the Galactic plane, the z dependence was averaged over as given below.

$$S(R,\phi)=\alpha\left[\int_{-Z_{max}}^{Z_{max}}\frac{\exp(-|z|/Z_0)}{S(R,z,\phi)}\mathrm{d}z\right]^{-1} \tag{3}$$

where, $\alpha=\int_{-Z_{max}}^{Z_{max}}\exp(-|z|/Z_0)\mathrm{d}z$, Z_{max} and Z_0 are taken to be 2·5 kpc and 0·45 kpc, respectively. The scale factors, $S(R,$f), computed for our sample range between 1 and ~ 6000 with 90% of the sample having scale factors less than 500. The large values of the scale factors can be in error by a large factor as in such cases the pulsars are situated most often in the inner region of the Galaxy where modelling of the selection effects is not very satisfactory. Also, if only a small fraction of the sample has a very wide tail in the scale factor distribution, then that fraction would dominate in the analysis and result in poor statistics. Hence, we have excluded about 10% of our sample of pulsars (those with scale factors > 500) from the sample used for rest of the analysis presented in this paper. This automatically excludes the pulsars within a radius of 4 kpc from the Galactic centre, this is also the region where the spiral arms are poorly defined.

3. Correlation between the pulsar distribution and the spiral arms

The relative galactocentric angular motion between the matter in the Galaxy and the spiral density wave pattern in a given time t can be given by (see Lin *et al.* 1969),

$$\beta(t,R)=V_{rot}\left(\frac{1}{R}-\frac{1}{R_c}\right)t \tag{4}$$

where V_{rot} is the velocity in the model of the Galaxy with a flat rotation curve (we assume the IAU recommended value of 225 km/sec. See Kerr & Lynden-Bell 1986) and R_c is the corotation resonance radius of the Galaxy. At this radius, circular velocity of the spiral pattern and the circular velocity of the matter around the centre of the Galaxy are equal. Matter leads the spiral pattern inside the corotation radius, and will lag behind outside. This relation enables us to find out the distribution of the pulsar population relative to the spiral arms at any given epoch (present epoch being taken as the reference). If pulsars had no space velocities, then their present positions should correlate with the mass distribution at the epoch of the formation of their progenitors. This correlation is, however, expected to be smeared due to: (1) The spread of the birth places of the progenitors, and (2) The velocity of the progenitors. But the star forming regions are expected to be confined within a belt along the spiral arms of width ≈ 100 pc. As the progenitors have typical velocities of about 15 km/sec (see Gies & Bolton 1986; Lequeux 1979), this would cause a smearing of few hundreds of parsecs in space. Indeed, if one can establish that the present distribution traces the location of the spiral arms at a past epoch, then it should give us useful constraints on the parameters related to the above effects.

Before proceeding further, for convenience we make a provisional assumption that the mass distribution in the spiral pattern is adequately described by the 'arm component' of the electron distribution model of Taylor & Cordes (1993). As this electron density model is based on the observations of Giant HII regions (see Taylor & Cordes 1993; Georgelin & Georgelin 1976; Caswell & Haynes 1987 for details), we consider our assumption reasonable. Our second assumption is about the value of the corotation radius (R_c) which gives a normalization for the rotation rate of the spiral pattern. We have assumed a value of 14 kpc for the R_c since this seems to fit the HI data rather well (Burton 1971). As we will see in one of the following sections (section 3.2), this value for the corotation radius is quite reasonable. Given these assumptions, we define $C(t)$ a measure of the correlation between the pulsar distribution in the Galaxy and the spiral pattern by,

$$C(t) = \frac{\sum_{i=1}^{N_{\rm psr}} S(R_i, \phi_i) n(R_i, \phi_i - \beta(t, R_i))}{\sum_{i=1}^{N_{\rm psr}} S(R_i, \phi_i)} \tag{5}$$

where, n is the mass density at the extrapolated location of i^{th} pulsar and $S(R_i, \phi_i)$ is the corresponding scale factor. The mass distribution perpendicular to the arm in this model is gaussian with $\sigma = 0{\cdot}3$ kpc and the width of the arm is restricted to 3σ on either side. This correlation is computed for a given relative epoch and corotation resonance radius, by rotating the pulsar distribution relative to the spiral pattern as per equation (4). The variation of the correlation as a function of the relative epoch for an assumed value of 14 kpc for the corotation resonance radius is shown in Fig. 4. *This plot shows a peak correlation at an epoch corresponding to about 60 Myr ago* accompanied by another peak around the present epoch. It is the correlation at

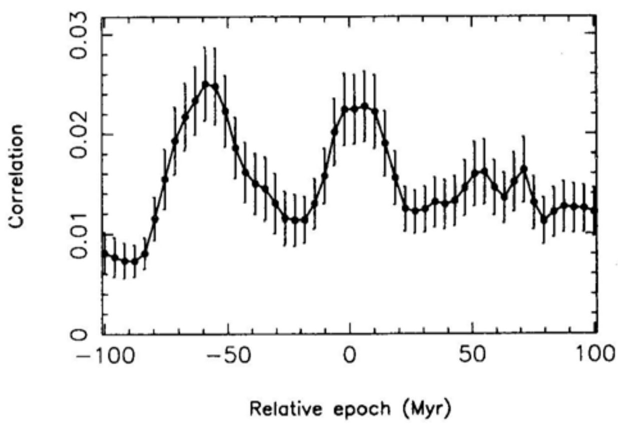

Figure 4. Plot of the correlation of the mass distribution in the spiral pattern at various epochs with the present pulsar distribution.This plot corresponds to a corotation resonance radius of 14kps.The error bars indicate 1σ derivation on either side.The locations of the peaks of the correlation correspond to an epoch which is 60 Myr ago and the present epoch.

– 60 Myr that we wish to identify as being associated with the past positions of the spiral pattern traced by the present distribution of the pulsars. We will discuss in section 5 some possible causes for the secondary peak in the correlation at the present epoch.

We have excluded from the above analysis pulsars with magnetic fields in the range $10^{10} < B < 10^{11.5}$ gauss as they have been tentatively identified as a separate population of recycled pulsars from intermediate mass binary systems (Deshpande *et al.* 1994). The correlation peaks seen in Fig. 4 are not affected even if these pulsars are included.

3.1 *Significance of the Correlation Maximum*

To test the significance of the above mentioned correlation maxima we adopted the following procedure: (1) Every pulsar was randomly assigned the longitude of some other pulsar, keeping their original distances the same, (2) Scale factors corresponding to their new positions in the Galaxy were computed, (3) This new set of parameters were used to compute the correlation as a function, of the relative epoch (using equation 5) and the value of the maximum correlation was noted.

The above mentioned procedure (step (1) to step (3)) was repeated thirty thousand times. Then, from the distribution of maxima of correlations, we find that the maximum found at the relative epoch of – 60 Myr with the original data (in Fig. 4) has a significance of 99·95%.

After scrambling the longitudes, a further test was made by also varying the distances by about 30% (rms), which showed an even higher significance for the correlation maximum found at – 60 Myr.

The significance of the correlation feature around the present epoch (0 Myr) was also tested in a similar manner but with maxima searched over an epoch range of – 25 Myr to 25 Myr. The significance of this feature is found to be only 93·3% which is rather poor.

3.2 *Resonance Radii of the Galaxy*

As mentioned earlier, Fig. 4 shows a peak correlation for a corotation radius (R_c) of 14 kpc and an epoch corresponding to about 60 Myr ago. We have repeated the correlation analysis for various values of the corotation radius. We find that our sample being mostly confined within a radius of 13 kpc from the Galactic centre is, naturally, not very sensitive to variations in the corotation radius beyond about 12 kpc. However, when the value of R_c is reduced below 12 kpc, the correlation peak (at – 60 Myr) broadens and shifts rapidly to earlier epochs disappearing completely for R_c below about 10 kpc. This is consistent with the observations of external galaxies where in all cases the pattern is found to be trailing the gas as far as the matter can be seen.

The resonance radii (Inner and Outer Lindblad resonance radii and the corotation resonance radius) play a central role in the study of spiral patterns and bars in galaxies. The relation between the resonance radii is as given below (see Binney & Tremaine 1987).

$$\omega_{\mathrm{ILR}} = \omega_p + \kappa/2; \qquad \omega_{\mathrm{OLR}} = \omega_p - \kappa/2 \tag{6}$$

Here, ω_p is the spiral pattern angular velocity, κ is the epicyclic frequency, $\omega_{\rm ILR}$ is the angular velocity corresponding to the inner Lindblad resonance and $\omega_{\rm OLR}$ is the angular velocity corresponding to the outer Lindblad resonance. For a flat rotation curve, ($\kappa = \sqrt{2}\omega$). Thus, for a given value of the corotation radius R_c, the inner and the outer Lindblad resonance radii would be about 0.3 R_c, and 1.7 R_c, respectively. Since there is clear evidence of spiral arms beyond ~ 4 kpc, the inner Lindblad resonance radius is unlikely to be larger than this. This would suggest a value for the corotation radius ⩽ 13 kpc, consistent with what we find from our analysis viz. $12 \leqslant R_c \leqslant 15$ kpc. Thus, our estimate for the inner and outer Lindblad resonance radii are in the range 3.5–4.5 kpc, and 21–25 kpc, respectively. (We wish to mention in passing that Mulder & Liem (1986) estimate R_c, to be ~ 8.5 kpc from their global gas dynamical model.)

3.3 *Minimum Mass for Neutron Star Formation*

In his paper Blaauw (1985) restricted himself to the observed sample of pulsars whose distances from the sun projected on to the plane of the Galaxy are within 0.5 kpc. After calculating the deathrate of massive OB stars in the OB associations in the solar neighbourhood, he arrived at the conclusion that the local pulsar population cannot be replenished by the OB associations alone. The field stars must therefore make significant contribution. While trying to match the local pulsar birthrate with the local deathrate of massive stars, he concludes that the overwhelming majority of the pulsar progenitors should correspond to relatively less massive stars (in the range 6–10 $M_\odot$). Based on more limited data Gunn & Ostriker (1970) had also come to the conclusion that the estimated pulsar birthrate implied relatively low mass for their progenitors. If this is true, then since the less massive stars live longer, an equally important corollary is that 'pulsars, on the Galactic scale, are tracers of past spiral structure (20–50 Myr ago) rather than of active spiral structure' (Blaauw 1985).

Our correlation analysis confirms this prescient conjecture. As can be seen from our correlation curve in Fig. 4, the correlation peaks at ≈ – 60 Myr. This means that the epoch of the formation of stars which gave birth to most of the present population of pulsars was ≈ 60 Myr ago. To a first approximation this timescale is the sum of the lifetime of the progenitors of pulsars and the average age of the present population of pulsars. We estimate the average age of pulsars in the present sample as follows,

$$\langle \tau \rangle = \frac{\sum_{i=1}^{N'_{\rm psr}} S_i(R,\phi)\tau_i}{\sum_{i=1}^{N'_{\rm psr}} S_i(R,\phi)} \tag{7}$$

Here, τ_i is the characteristic age (defined as $P/2\dot{P}$) of i^{th} pulsar and $S_i(R,\phi)$ is the corresponding scale factor. $N'_{\rm psr}$ is the number of pulsars confined to the regions where the model mass density is non-zero at the relative epoch of – 60 Myr. This average age($\langle \tau \rangle$) turns out to be 10 ± 2 Myr. Therefore, the average lifetime of the progenitors is about 50 Myr, which would correspond to an 7 $M_\odot$ star (see Schaller *et al.* 1992). So, all stars with masses greater than about 7 $M_\odot$ should end up as neutron stars. We will discuss this conclusion in greater detail in section 5.

It is worth mentioning here that when we selected pulsars in different average age

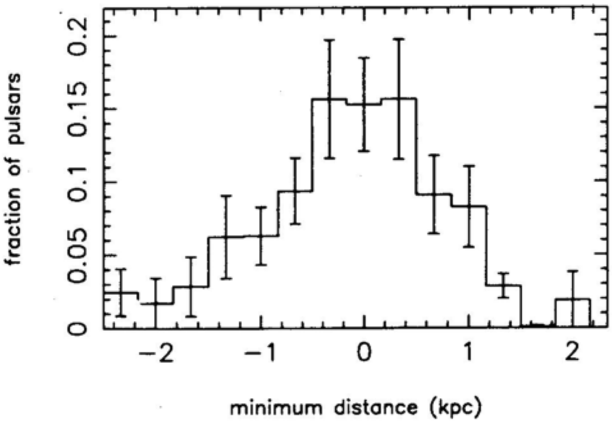

Figure 5. Distribution of d_{min} after compensating for the selection effects. Y-axis gives the *true* number of pulsars (after scaling the observed number of pulsars by appropriate scale factors as a function of position in the Galaxy). This distribution is produced after *rotating* the Galaxy by – 60Myr. The error bars indicate 1s deviation on either side.

ranges, the correlation peak (corresponding to the past epoch) was seen to shift systematically implying a roughly constant value for the lifetime of the progenitor stars.

3.4 *Modified* d_{min} *Distribution*

As mentioned earlier, the scale factor is the ratio of the *true* number of pulsars in the Galaxy to the observed number. If the Galaxy is rotated back to the epoch (– 60Myr) where the correlation is maximum, one expects to find the distribution of d_{min} with respect to the scale factors $S(R,\phi)$ symmetric and centred around zero d_{min}. This is one of the necessary conditions to check the estimation of scale factors. Fig. 5 shows this distribution after rotating the Galaxy back in time corresponding to – 60Myr. This distribution is reasonably symmetric in contrast to Fig. 2. We have also confirmed that this modified d_{min} distribution is more compact and symmetric than similar distributions computed for various other epochs.

4. Space velocities of pulsars

As explained in the last section, after correcting for the selection effects we get a symmetric compact number distribution as a function of d_{min}. If we assume the space velocities of pulsars as the prime factor causing the spread seen in the d_{min} distribution, it is possible to estimate some useful quantities related to these velocities.

For this purpose, we selected a subset of our sample that contributes to the peak of the correlation in Fig. 4. This subset therefore confines $|d_{\mathrm{min}}|$ to $\leqslant 0{\cdot}9$ kpc (as only in this d_{min} range the model mass density is assumed to be non-zero), and characteristic age τ_{ch} less than l0Myr. Also, while deriving the velocity distribution, we have neglected the pulsars with $\log B < 11.5$ as they would introduce a systematic bias towards low velocities. It can be argued that most of the pulsars with fields in the range $10^{10-11.5}$ are recycled pulsars from wide binaries which are expected to have low velocities (Bailes 1989; Deshpande et al. 1994). For each pulsar thus selected, a *projected velocity* can be computed by dividing its $d_{min,}$ value by its characteristic age.

The connection between the *projected velocity* distribution and the real velocity distribution involves the following arguments. If pulsars are distributed following a real velocity distribution $P(v)$ and evolved for a fixed age τ, the number of pulsars in a unit volume at a distance $(v\tau)$ is simply proportional to $P(v)/(v\tau)^2$. Pulsars in a shell of radius $(v_0\tau)$ contribute uniformly to all *projected velocities in* the range zero to v_0. In general, the distribution of *projected velocities* (v_p) *can* be expressed as,

$$P_p(v_p) \propto \int_{v_p}^{\infty} \frac{P(v)}{v} \mathrm{d}v \tag{8}$$

It is worth noting that as v_p is defined in the plane of the Galaxy, the above procedure is unaffected by the height of the birth places above the plane and by accelerations normal to the plane.

4.1 *Accounting for Distance Uncertainties*

Before one uses the $d_{min,}$ distribution to deduce a velocity distribution it is important to assess the effect of uncertainties in the assumed distances to pulsars on the estimation of d_{min}. Taylor & Cordes (1993), whose electron density model forms the basis of our distance estimation, have shown that in most cases the uncertainty in distance is < 25%. If the range of d_{min} values is comparable to the errors in the distance to pulsars from the sun, then the conclusions derived regarding the velocities of pulsars would be seriously affected (we thank J. P. Ostriker for pointing this out).

In order to independently assess the magnitude of the distance uncertainties relevant to the samples used in the present analysis, we have used the following procedure.

We add x% error to the distances and compute a distribution of maximum correlation (C) by a Monte Carlo simulation. For x small compared to the intrinsic error x_0, this should make little difference and the computed distribution will be highly peaked around C_0 (the maximum correlation in Fig. 4). For x large compared to the intrinsic error x_0, our ensemble will behave like the one in which the *true* distances have errors $x \gg x_0$. In this case the correlations C will be typically smaller than C_0 implying a higher significance for C_0. We estimate x_0, the intrinsic error in the model, by that value of x for which this significance saturates. We found that the significance level increased from about 40% when negligible extra error was introduced in distances, to about 80% when the extra error in distances was about 20%. Although this indicates that the probable distance error in our data is likely to be $\leqslant 20$%, to be conservative we have used the value of 20% error for further analysis. Incidentally, the estimated error in the distance derived by us from the model due to Taylor & Cordes (1993) agrees quite well with their estimate.

For every pulsar in our sample set we varied the distance by 20% (rms) and computed the resulting variance .$\sigma^2_{d_{min}}$ of d_{min}. The implied error in the *projected velocity* is then $\Delta v = \sigma_{d_{min}}/\tau_{ch}$.To reduce the contribution from pulsars with large values of Δv, we have weighted their contributions in the estimation by the following function.

$$w_d = \exp\left[-\left(\frac{\sigma_{d_{min}}}{\tau_{ch}\Delta v_{max}}\right)^2\right] \tag{9}$$

Here,Δv_{max} is arbitrarily chosen to be 40km/sec. We also estimated the resultant spread in the velocity distribution caused merely due to the distance uncertainties as a function of the tolerance level (Δv_{max}). For arbitrarily large Δv_{max} (which is the case where w_d is more or less the same for all pulsars), this resultant rms spread turns out to be about 500 km/sec (for about 20% error in distances), emphasising the need for the explicit weighting by w_d with a realistic tolerance level. With the choice of 40 km/sec for the tolerance level, the spread comes down to about 42 km/sec. Hence we believe that the *mean velocity* estimated using the above weighting function will not be seriously affected by the distance uncertainties, provided it is of the order of or greater than ≈ 80 km/sec.

4.2 *Aliasing due to Finite Interarm Spacing*

It should be remembered that the contributions from velocities greater than $(D/2\tau)$, where D is the interarm spacing, get *aliased* with the contributions in the range zero to $(D/2\tau)$. This effect is naturally more serious in the regions where the interarm spacing is small and can lead to underestimation of the mean velocity. We have tried to overcome this difficulty by appropriately weighting down the contributions from regions of severe aliasing by using the following weighting function,

$$w = \left[\frac{D}{D_0}\right]^a, \quad D < D_0$$
$$w = 1 \qquad D \geqslant D_0 \tag{10}$$

Here, D is the interarm spacing at the location of a given pulsar and a is a weighting exponent. We have used D_0 of 4 kpc and a weighting exponent of unity. In our sample, most of the pulsars which would be seriously affected by *aliasing* are also the ones that are weighted down while accounting for the distance uncertainties. Therefore, our results werefound notto be verysensitive to the parameters of this weighting function.

4.3 *Estimation of Average Velocity*

We have estimated a *projected velocity distribution* as well as the mean value of d_{min} (0.3 ± 0.07) and the mean age (2.8 ± 0.7) of pulsars whose contributions have been weighted appropriately to minimize the effect of distance uncertainties and possible aliasing as described above. Using the above average values, a simple minded estimation of the mean *projected velocity* turns out to be (105 ± 35) km/sec. If we assume a Maxwellian type distribution for the speed, then this implies a mean velocity of about (200 ± 70) km/sec. We have also estimated an average value of the *projected velocity* implied by the d_{min} and characteristic age combination considered for each pulsar contributing to the correlation. This average *projected velocity* is (80 ± 20)km/sec implying a mean velocity of about (160 ± 40) km/sec. In principle one should be able to do a detailed fit on the *projected velocity* distribution. However, in the present case, because of high statistical noise we are not in a position to carry out this exercise. Also, because we have chosen a set of pulsars which are within ± 0.9 kpc from the arms (see the beginning of section 4), the present analysis is not sensitive to high velocities (v > 400km/sec).

5.Conclusions and discussions

As described in the earlier sections, an examination of the correlation between the location of the spiral arm at different epochs and the present pulsar distribution has enabled us to estimate three important parameters: (1) The corotation resonance radius of the Galaxy, (2) The minimum mass of pulsar progenitors, and (3) The distribution of pulsar velocities.

The steps and assumptions involved in inferring the minimum mass of pulsar progenitors from our correlation analysis need a careful discussion. The Initial Mass Function of stars is believed to be steep ($N(M) = M^{-2.35}$, Salpeter 1955). Hence, it is reasonable to assume that the majority of pulsars come from progenitors with masses just above the cutoff mass for the formation of neutron stars. As we do not expect any particular correlation of the lifetime of pulsars with the mass of the progenitors, the above assumption should hold good. Therefore, the epoch of maximum correlation (Fig. 4) should correspond to the formation of progenitors close to the cutoff mass. Reduced correlations are expected from pulsars formed from more massive progenitors which should show up at more recent epochs (than the epoch corresponding to correlation maximum) consistent with the evolution timescales of the progenitors and the average age of the corresponding pulsars. A significant tail corresponding to such more recent epochs is clearly seen in Fig. 4.

While associating the inferred evolution timescale (≈ 50 Myr) with a mass of the progenitor ~ 7 $M_\odot$, we make an implicit assumption that the progenitors are *single stars.* If the progenitors with the above evolution timescales were to be in binaries, the progenitors need to be more massive. This is because we expect the time interval between the star formation and the release of an 'observable' pulsar to be longer by as much as a factor of two or more (depending on the mass ratio) compared to the evolution timescales of the progenitors if they were single stars. Conversely, the single star fraction of the progenitors of a given mass will contribute collectively to the correlation over a narrow range of epochs (typically over the average age of pulsars), while the remainder will contribute to the correlation at epochs spread over a large range depending on the mass of the companion. Therefore, the fact that the correlation seems to be peaking over a narrow range of epochs should necessarily be interpreted as due to the contribution from 'single' progenitors.

A formal uncertainty in the epoch of maximum correlation can be estimated from the statistical errors in the correlation (shown in Fig. 4). This uncertainty is estimated to be about ± l0Myr. However, most of it can be attributed to a genuine width of a correlation function that one would obtain for a fixed mass of progenitors (for example, if stars of the limiting mass are born over a region of width ≈ 100 pc along the spiral arms, one would see the correlation width to be 16 Myr). Hence, we believe that the uncertainty in estimating the epoch of maximum correlation is small enough to take seriously our conclusion that all stars above 7M must produce pulsars.

The correlation between the pulsar distribution and the spiral pattern as a function of azimuthal rotation is sensitive in the regions where the arms are not azimuthal and in the regions away from corotation. The pulsars in the region of corotation and the regions where the spiral arms are predominantly azimuthal, contribute to a non-varying component of the correlation as can be clearly seen in Fig. 4. We have confirmed this behaviour by excluding the contribution from such regions.

While computing the correlations at various epochs we have used only the azimuthal shifts appropriate to the *present* galactocentric radius of pulsars as their galactocentric distances at earlier epochs cannot in general be guessed. However, the error introduced due to this should get averaged out as our sample is sufficiently large.

The origin of the feature at the present epoch (see Fig. 4) is not entirely clear. Such a feature, however, is expected to appear if the electron density model used for estimating the dispersion measure distances to pulsars somehow on the average tends to artificially cluster pulsars in the spiral arms. One possible way this can happen is if the ratio of electron density in the *arm component* to that in the *smooth component* has been overestimated in the electron density model. Another possibility is that the arm pattern in the model deviates in some systematic way from the *true* pattern. In such a case the artificial clustering of pulsars in the arms should be more pronounced in the inner regions of the Galaxy where the model is likely to be inadequately constrained. We have looked at the correlation function from pulsars confined to the inner part of the Galaxy (galactocentric radius less than 7 kpc) and find the 0 Myr feature more pronounced. As a preliminary test we varied the model electron density in the arm by factors on either side of unity, recomputed the pulsar distances and found that the relative strength of the 0 Myr feature (with respect to the – 60 Myr feature) *dropped* when the arm density was reduced by about 10%, and *increased* when the arm density was increased by about 10%. The real explanation for the origin of the 0 Myr feature may, however, involve a combination of the above mentioned possibilities in addition to some others that we have not considered here. We wish to emphasize that the – 60 Myr feature is seen clearly in all the subsets of the data (e.g. in different ranges of the galactocentric radii; with different scale factor cutoffs; different characteristic age groups; etc.). This and the high statistical significance associated with the correlation peak at – 60 Myr (as discussed in section 3.1) suggests that the validity of the correlation maximum found at – 60 Myr is not affected seriously by the possible causes for the 0 Myr feature.

While estimating the average pulsar velocity, we have ignored the spread in the $d_{\min}$ distribution due to the width of the spiral arms (over which star formation occurs) and the progenitor velocities. Since pulsar velocities are quite large compared to the expected velocities of their progenitors (see Lyne *et al.* 1982; Bailes *et al.* 1990; Harrison *et al.* 1993), we expect the former to dominate the spread in the $d_{\min}$ distribution. Also, while deriving the velocity distribution, we have neglected the pulsars with log $B < 11.5$ as they would introduce a systematic bias towards low velocities. This is because most of these pulsars with fields in the range $10^{10} – 10^{11\cdot5}$ gauss have been identified as recycled pulsars and have been shown to have very low velocities (Bailes 1989; Deshpande *et al.* 1994) due to their binary history.

The velocities inferred from VLBI proper motion measurements are on the average much higher than the mean velocity of 160–200 km/sec as estimated by us in section 4. If the large proper motions are considered as part of the tail of our velocity distribution then there should be a large number of low velocity pulsars for which such proper motion measurements are yet to be done. This is not very surprising since the proper motion measurement techniques tend to select against low velocity and far away pulsars.

From an independent analysis of the pulsar population, Narayan & Ostriker (1990) have argued for two distributions for the pulsar velocities. They estimate 50 and 150km/s as the mean one-dimensional velocities for their low and high velocity distributions respectively with roughly half of the pulsar population in each distribution.

This would imply a mean velocity of the population that is in agreement with our estimate (≈ 160–200 km/sec).

Summary

The main results of this paper can be summarized as follows:

1. The corotation resonance radius of the Galaxy is estimated to be (13.5 ± 1.5) kpc. Assuming a flat rotation curve for the Galaxy, the other two resonance radii (the inner and outer Lindblad radii) are estimated to be in the range 3.6–4.5 kpc and 21.4–25.5 kpc, respectively.
2. The correlation between the present position of pulsars and the location of the spiral pattern in the past suggests a mean lifetime of their progenitors ≈ 50 Myr. This would imply that all stars more massive than approximately 7 $M_\odot$ must leave behind neutron stars.
3. The global mean of pulsar velocities is estimated to be in the range ≈ 160–200 km/sec.

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